Multiplicity Correlation of Prompt Neutrons and Gammas in the Spontaneous Fission of Cf-252

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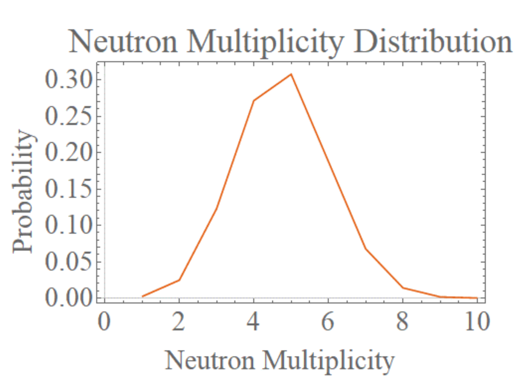
Introduction

Neutrons and photons are emitted during the fission process, when a heavy nucleus splits into two energetically excited fragments. In the de-excitation process, multiple prompt neutrons and photons are emitted in coincidence, see Fig. 1. A possible dependence, between neutron and photon multiplicity is investigated by analyzing collected data. A negative dependence, or a competition, is expected between neutrons and photons. An array of 45 liquid organic scintillation detectors and a fission chamber are used to detect neutrons and photons multiplets fission-by-fission. Detector response dominates the detected data, and an unfolding technique has to be applied in order to extract useful correlation information.



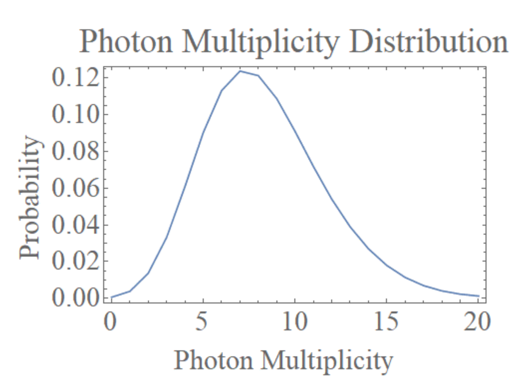


Figure 1: Multiplicity distribution of CF-252 for prompt neutrons and photons, from Ref. 1. Understanding the correlation between these two distributions is the objective of this work.

The problem

The relationship between prompt neutrons and photons emitted immediately after a fission event is investigated. In order to quantify this relationship, the Pearson correlation coefficient is used. The Pearson correlation coefficient at emission $\mathrm{corr}(n,p)$, which measures the linear relationship between two variables, is computed from:

$$\mathrm{corr}(n,p) = \frac{E[n,p]-E[n]E[p]}{\sigma[n]\sigma[p]}, \tag{1}$$

where $E[n]$ and $E[p]$ are the first central moments, the averages, of the neutron and photon multiplicity distribution, respectively; $\sigma[n]$ and $\sigma[p]$ are the second central moments, or the standard distributions and $E[n,p]$ is the joint neutron-photon first moment.

The result, the correlation coefficient, is a unitless measure that ranges from +1 to 1, indicating respectively positive and negative correlation. A correlation of 0 indicates no correlation between the variables. This quantity requires the knowledge of the following:

- First and second moments of the multiplicity distribution of neutrons and photons
- Joint first moment of the two distributions

The purpose of this work is to find expressions for the moments of the physical emission, given the data collected.

Measurement

- 45 organic liquid scintillators were used to measure prompt neutron and photons from a Cf-252 spontaneous fission source, as shown in Fig. 2.
- A Cf-252 fission chamber signal was used to time-correlate particle detections.
- 1 m Cf-252 source-detector distance for all detectors.
- Pulse shape discrimination was used to classify neutron and photon-ray detections in the liquid organic scintillators.
- The system has a small absolute efficiency, 2-3%, which significantly influences the observed multiplicity distribution at detection.

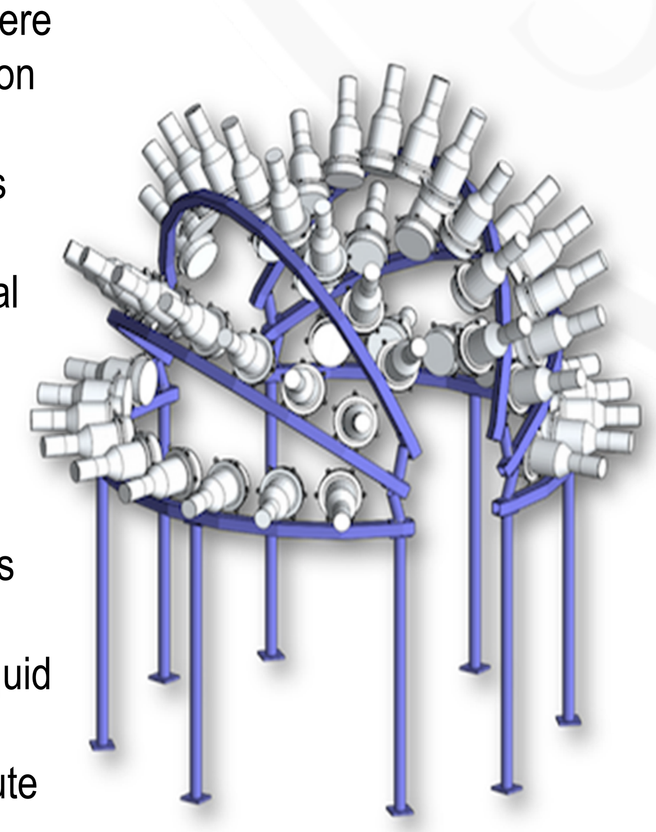


Figure 2: A model of the detector used to measure the Cf-252 spontaneous fission source.

Data Collected

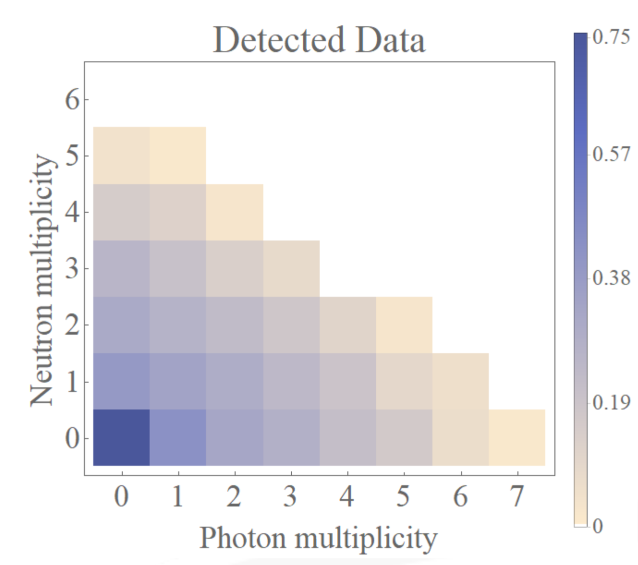


Figure 3: Measured multiplicity distribution. Each cell corresponds to a neutron-photon multiplicity coincidence

After collecting data for 10^8 fission events, a neutron-photon probability distribution of detection data is inferred, visualized in Fig. 3.

Due to the low efficiency of the system, much of the original multiplicity information is "lost". It is however still possible to reconstruct some of the features of the original distribution.

Data Analysis

An unfolding technique is an algorithm that attempts to solve the linear equation:

$$\hat{\boldsymbol{R}}\,|n\rangle = |d\rangle\,, \tag{2}$$

where $|n\rangle$ is the probability distribution vector at emission, $|d\rangle$ is the probability distribution vector from the measured data and $\hat{\boldsymbol{R}}$ is the response of the system, which incorporates absolute efficiency and losses of the measurement setup. While $|d\rangle$ is directly observed, $\hat{\boldsymbol{R}}$ has to be estimated, either analytically or by simulation. The goal of this unfolding is to find the emission probability $|n\rangle$.

Inverse

Inverse solutions seek to solve (2) by inverting the response matrix:

$$\hat{R}^{-1}\,|d\rangle = |n\rangle\,, \tag{3}$$

which yields exact solutions if the response matrix is known accurately. For low absolute efficiency of the system, the inverse of the response is extremely unstable and does not yield good results.

Forward

Forward solutions seek to solve (2) by assuming a general known form of the emission with free parameters $|\tilde{n}\,(k_i)\rangle$

$$\hat{R}\,|\tilde{n}\,(k_i)\rangle = |\tilde{d}\,(k_i)\rangle, \tag{4}$$

the parameters k_i for which the parametrized detection $|\tilde{d}\,(k_i)\rangle$ matches the real detection $|d\rangle$ are the solutions to this problem.

Moments Method

Neither techniques, inverse or forward, are suitable to solve the problem considered. The former is too ill-posed while the latter requires knowledge of the answer that we seek. If the efficiency ϵ of the system is assumed to be independent of multiplicity and detection, the response may be written as:

$$\hat{R} \approx \binom{n}{d}\epsilon^d(1-\epsilon)^{n-d} = \hat{R}(\epsilon), \tag{5}$$

where the factor in parenthesis is the binomial coefficient; for this reason the response is called "Binomial Response". The factorial moments f_j of a distribution furnish an excellent basis to work in for the binomial response. The factorial moment of order *j* is a specific linear combination of the moments up to the same order. The set of the factorial moments of a distribution provides the eigenfunctions of the response:

$$\hat{R}(\epsilon)f_j = \epsilon^j\,f_j. \tag{6}$$

This expression, used in combination with (2), relates the factorial moments of emission and detection.

2-Dimensional Moments

It is possible to generalize (6) to factorial moments of probability distributions of two variables, neutrons and photons in our case:

$$\hat{R}(\epsilon_n,\epsilon_p)f_{j,k} = \epsilon_n^j\epsilon_p^k f_{j,k}, \tag{7}$$

where ϵ_n and ϵ_p are the total efficiencies of the system for neutrons and photons, respectively. The two-variable factorial moment $f_{j,k}$ is of order *j* in the neutrons distribution and of order k in the photons distribution.

Application

By knowing the relationship between the central moments and the factorial ones, and applying (6) and (7), it is possible to find the correlation of emission, (1), in terms of the correlation in the measured distribution:

$$\mathrm{corr}(n,p) = \frac{\mathrm{corr}(d_n,d_p)}{\sqrt{\left(1-\frac{E[d_n]}{\sigma^2[d_n]}(1-\epsilon_n)\right)\left(1-\frac{E[d_p]}{\sigma^2[d_p]}(1-\epsilon_p)\right)}}\,, \tag{8}$$

which can now be written in terms of the more common central moments. Here d_n and d_p are the detected distribution of neutron and photon multiplicity, respectively. For very low efficiencies, the proportionality factor between emission and detection correlation reduces to $(\epsilon_n\epsilon_p)^{-1/2}$.

Results

The neutron and photon multiplicity correlation in the detected data is found to be $\mathrm{corr}(n,p) = -0.0091 \pm 0.0024$. By inverting the problem using the moments of the distribution, it is found that the sample correlation at emission is $\mathrm{corr}(n,p) = -0.34 \pm 0.11$There are many factors that are not considered here, for example multiplicity-dependent energy distributions and the unique response of individual detectors. The result in this form is not refined enough to provide an accurate estimate of the real correlation; this result is however statistically significant in showing that the correlation at emission is negative.

Future Work

In order to refine the results derived in the previous section, a few steps can be taken:

- Improve the quality of the simulation, to reduce the error in ϵ_n and ϵ_p.
- Apply correction factors to the response (5):
  $$\hat{R} = \hat{R}(\epsilon) + \alpha\,\hat{C}, \tag{9}$$
  where $\hat{C}$ is a correction to the binomial response, and α is a small number ($\alpha \ll 1$). Factorial moments are not eigenfunctions of this response; it is however possible to find the new eigenfunctions by perturbation.
- Repeat measurements with higher detector system efficiency.

Conclusion

In this ongoing work, we have developed an unfolding technique that may be used to extract the physical correlation of a pair of variables from measured data. The results, when applied to the detection data of prompt neutron and gammas of Cf-252, yields a relatively strong negative correlation. The exact numerical value of the correlation might have been affected by several parameters not yet considered, for example:

- Dead time in detector
- Energy dependence
- Angular dependence

which may affect the correlation significantly. The result does however suggest that a negative correlation, or competition, between prompt neutrons and photons exists.

Bibliography

[1] J. M. Verbeke et al, *"Simulation of Neutron and Gamma Ray Emission from Fission and Photofission"*, 2014

This work was funded in-part by the Consortium for Verification Technology under Department of Energy National Nuclear Security Administration award number DE-NA0002534

National Nuclear Security Administration