Quickest Change Detection in Nuclear Fuel Cycles

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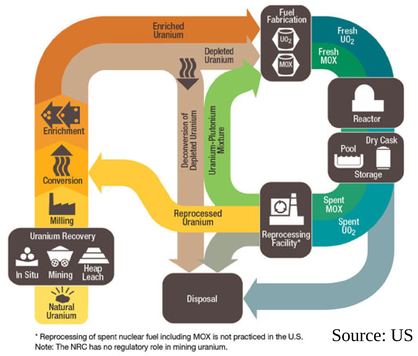
The Nuclear Fuel Cycle



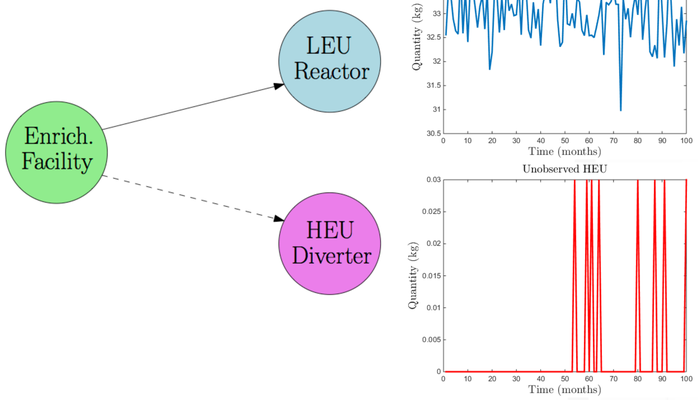
Source: US NRC

The basic objective of IAEA safeguards is

"...the *timely detection* of any diversion of significant quantities of nuclearmaterial and to deter diversion by creating the risk of *early detection*."

Leonard Weiss,
Bulletin of Atomic Scientists, vol. 47, no. 4, 1991.

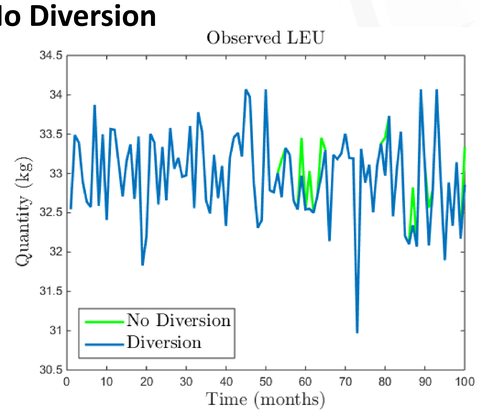
Simplified Model



- Data generated using CYCLUS, the nuclear fuel cycle simulator of the UW group [1].

Diversion vs. No Diversion

NOTE: Hard to detect Diversion by visual inspection

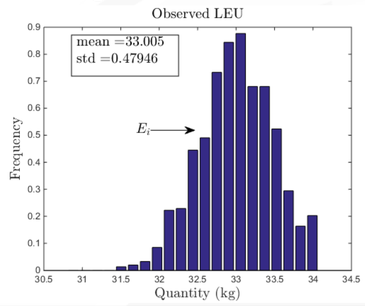


Inspired by: M.B. McGarry, GSWebinar_20150430 [1]

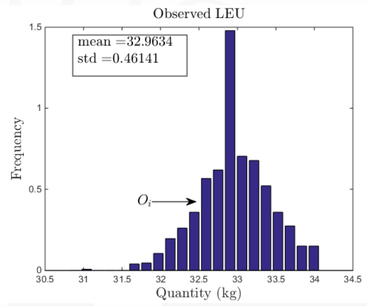
Statistical Hypothesis Testing: Offline Detection

H_0 : No diversion
H_1 : Diversion after $\boldsymbol{\tau}$ (Assume $\boldsymbol{\tau}$ given by oracle)

Histogram before τ



Histogram after τ



Chi-Squared Test [2]

- Difference between histograms before and after $\boldsymbol{\tau}$: significant or not?

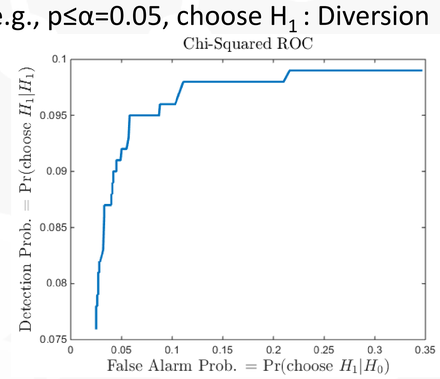
# of bins; observed count in i-th bin; expected count in i-th bin; Test statistic

$$S = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$$

Chi-squared distributed with dof N-1

$$p = \Pr(\chi^2_{N-1} \geq S)$$

- If p-value significantly small, e.g., p≤α=0.05, choose H_1 : Diversion
- Different α gives different (P_D,P_{FA}) pair
- Good but offline and needs an oracle



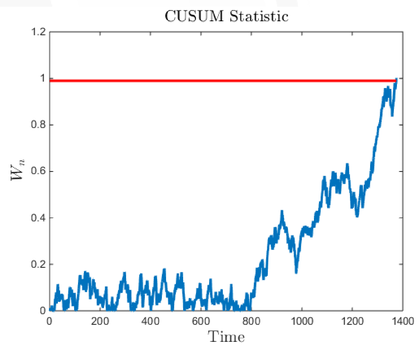
Quickest Change Detection: Online Detection

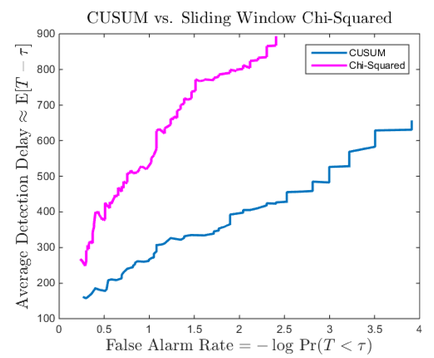
- Observe LEU quantity sequentially: $X_1, X_2, \ldots, X_n, \ldots$
- Assume a baseline probability distribution $f_0, e.g., N(\mu_0, \sigma_0)$ for no-diversion,
  and at unknown time $\boldsymbol{\tau}$ diversion begins and probability distribution changes to $f_1, e.g., N(\mu_1, \sigma_1)$
- At each time n, decide whether change occurred or not, and stop at time $\boldsymbol{T}$ when change is detected
- Cumulative Sum (CUSUM) algorithm [3] minimizes expected detection delay $\mathrm{E}[T - \tau]$ while keeping the false alarm rate restricted

$$W_n = \max\left\{W_{n-1} + \log\frac{f_1(X_n)}{f_0(X_n)}, 0\right\}$$

$$T = \min\{n: W_n > A\}$$

- Estimate unknown parameters or apply nonparametric CUSUM [4]





- Window size = 400 for chi-squared. Histogram in second half (suspected) is tested against the histogram in first half (baseline). If no change, window is moved by one and retested.

References

[1] CYCLUS, http://fuelcycle.org/
[2] A. Agresti, *Categorical Data Analysis*, 3rd Ed., Hoboken, NJ: Wiley, pp. 18-22, 2013
[3] V. Poor and O. Hadjiliadis, *Quickest Detection,* New York, NY: Cambridge, pp. 132, 2009
[4] T. Banerjee, H. Firouzi and A. Hero, *Non-parametric quickest change detection for large scale random matrices*, ISIT, Hong Kong, 2015.

Acknowledgements

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