

Multi-Centrality Graph Spectral Decompositions and Their Application to Cyber Intrusion Detection

Dr. Pin-Yu Chen¹

Dr. Sutanay Choudhury²

Prof. Alfred Hero¹

¹Department of Electrical Engineering and Computer Science
University of Michigan

²Pacific Northwest National Laboratory



Motivation

“The IAEA has reported cases of random malware-based attacks at nuclear plants”[1]

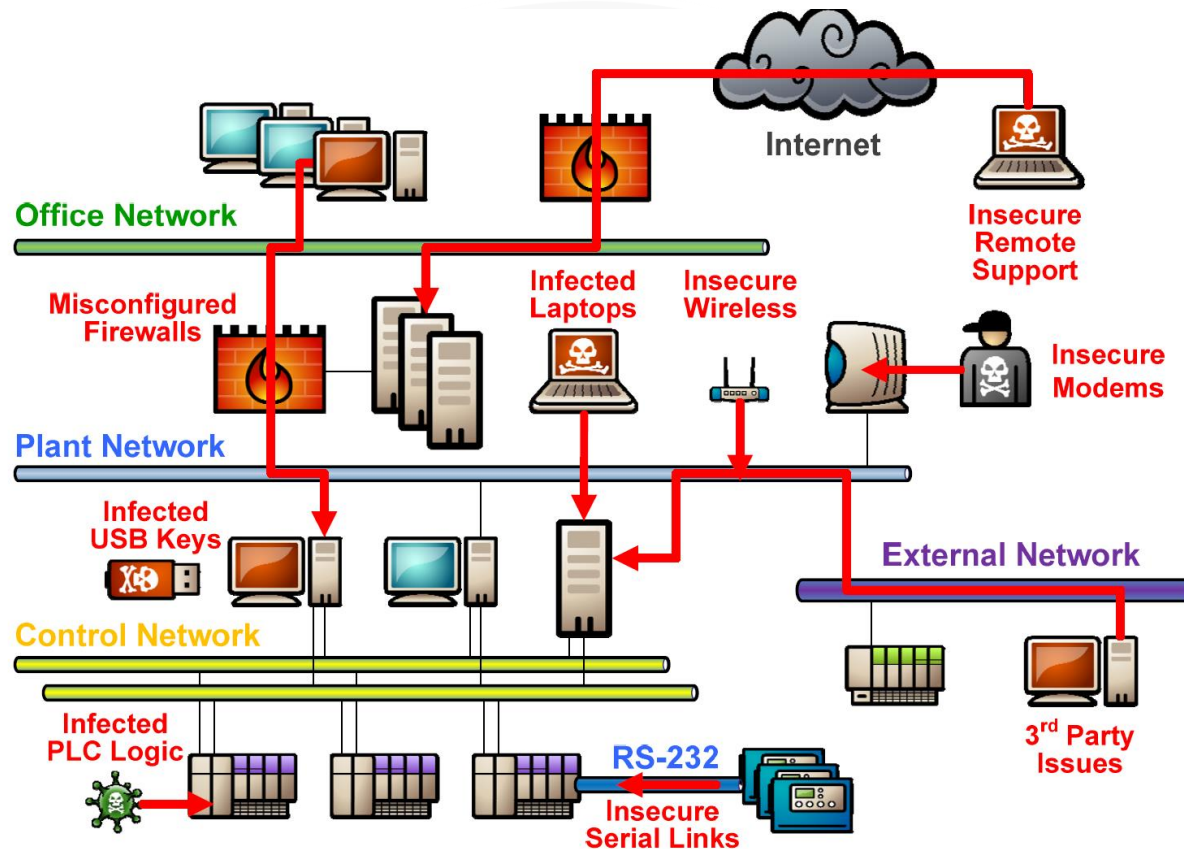
“Cyber threats to nuclear materials, nuclear facilities and nuclear command, control and communications are becoming more sophisticated every day, and the global technical capacity to address the threat is limited.”[2]

[1] Inter Press Service News Agency, Aug 17, 2015

[2] Senator Sam Nunn and Ted Turner, Nuclear Threat Initiative (NTI), 2016



Nuclear Facility Cyber Intrusion



Possible Pathways into a Control System

Source: Fig. 1, Eric Byres, Tofino Security White Paper, , 2012.

Cyber Security via Graph Mining

- Represent pairwise interactions between entities on network as a graph
- Extract structural features from a graph (or ensembles of graphs) for graph connectivity summarization and anomaly detection
- Transform graph representation to feature matrix representation
- Identify high vulnerability nodes
- Early detection of anomalies and attacks

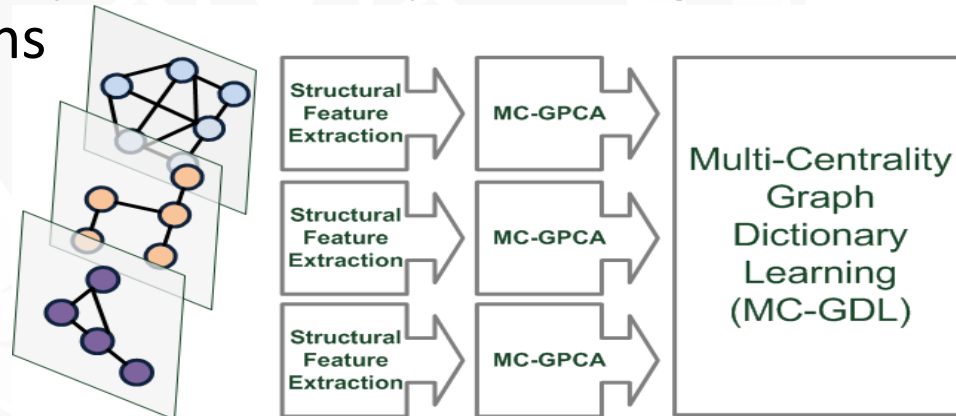


Our Contributions

- Multi-centrality graph PCA (MC-GPCA) for a single graph



- Multi-centrality graph dictionary learning (MC-GDL) for ensembles of graphs



- Application to cyber intrusion detection

Ref: Chen-Choudhury-Hero,

[ICASSP'16](#)

Graph Structural Feature Extraction

- **Goal:** extract structural features from a graph G of n nodes and represent them by an $n \times p$ feature matrix \mathbf{X}
 - p : # of extracted structural features
 - $\mathbf{X} = [\mathbf{x}_{\# \text{ } h\text{-hop walk}} \quad \mathbf{x}_{h\text{-hop weight}} \cdots \mathbf{x}_{\text{deg}} \quad \mathbf{x}_{\text{eig}} \quad \mathbf{x}_{\text{betweenness}} \cdots \mathbf{x}_{\text{ref}} \cdots]_{n \times p}$
- Three types of graph structural features are extracted:

(1) graph walk statistics

- # of h -hop walks – $\mathbf{a}^{(h+1)} = \mathbf{A}\mathbf{a}^{(h)}$
A: adjacency matrix of G
- total weight of h -hop walks – $\mathbf{w}^{(h+1)} = \mathbf{W}\mathbf{a}^{(h)} + \mathbf{A}\mathbf{w}^{(h)}$
W: edge weight matrix of G
- > efficient recursive computation

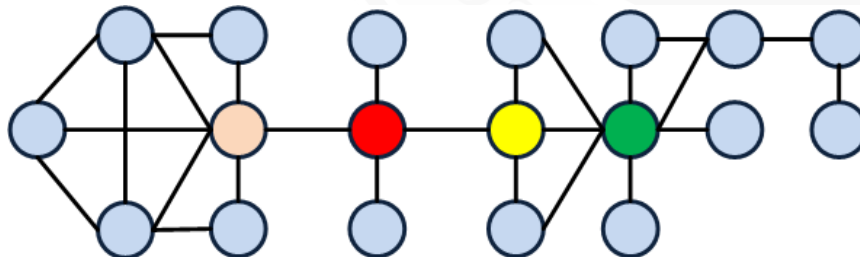


Graph Structural Feature Extraction

(2) centrality measures

- One centrality measure of each node = One extracted structural feature (one column of \mathbf{X})
- LFVC: local Fiedler vector centrality [Chen-Hero ICASSP'14, Comm. Mag.'14, Tran. Signal Processing'15]

Degree	• # of connections
Betweenness	• # of shortest paths
Closeness	• average hop distance
Eigenvector	• relative importance
Ego	• local betweenness
LFVC	• algebraic connectivity

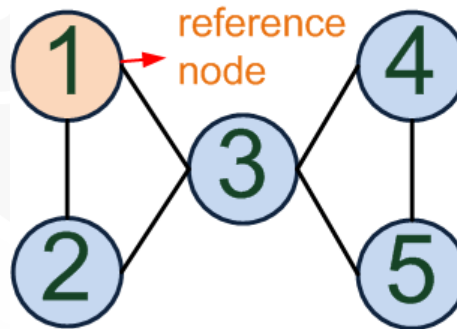


Graph Structural Feature Extraction

(3) hop distance to a set of reference nodes

- select some “anchor nodes” in the graph and use hop distances to these nodes as graph structural features
- enhance structural identifiability for graphs of high symmetry
- example:

- nodes 1,2,3,4 are indistinguishable using degree feature
- use node 1 as a reference node, then only nodes 4 and 5 are indistinguishable



$$x_{\text{deg}} = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 2 \\ 2 \end{bmatrix} \quad x_{\text{ref}} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

Multi-Centrality Graph PCA (MC-GPCA)

- Project (normalized and centered) structural feature matrix \mathbf{X} onto a set of uncorrelated orthogonal basis
- Advantages:
 - 1) Decorrelation – structural features are often correlated
 - 2) Visualization via projection coefficients

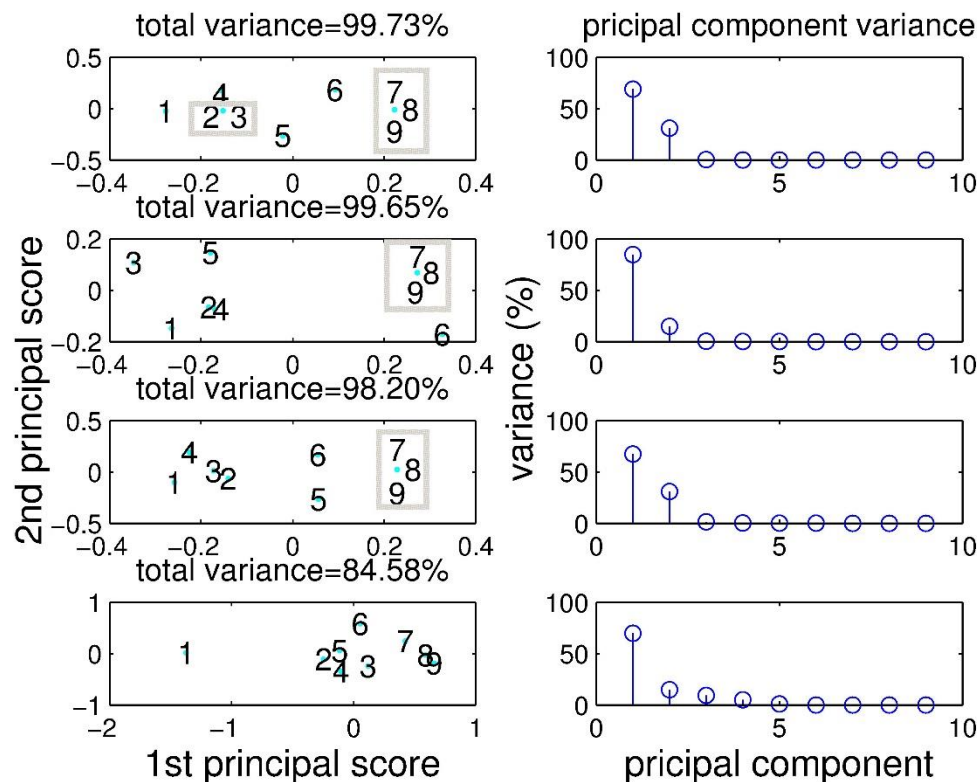
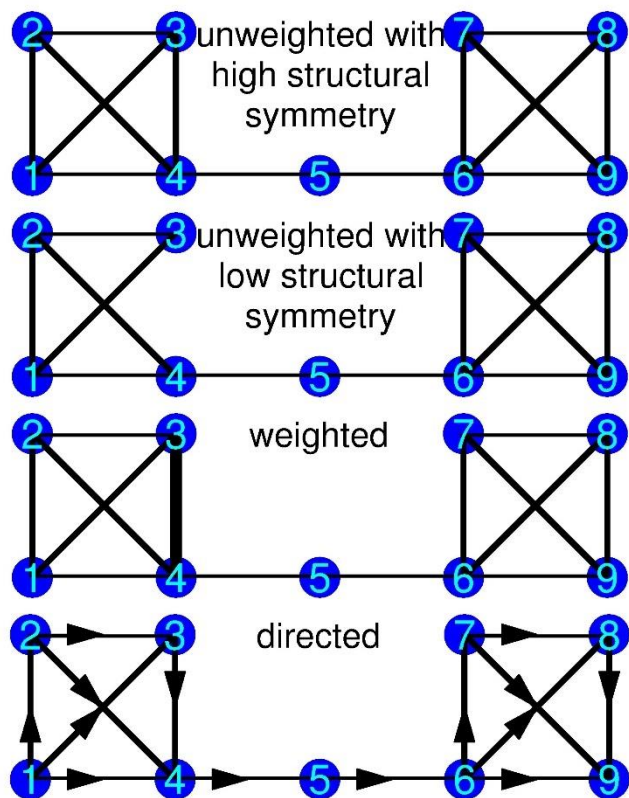
Algorithm 1 Multi-centrality graph PCA (MC-GPCA)

Input: A graph $G = (\mathcal{V}, \mathcal{E})$, desired dimension q

Output: n structural coordinates \mathbf{Y} for each node in G

1. Extract p structural vectors \mathbf{X} from G
 2. Normalize each column of \mathbf{X} to have unit norm
 3. Subtract row-wise empirical average from \mathbf{X}
 4. Solve the right singular vectors \mathbf{V}_q of \mathbf{X}
 5. $\mathbf{Y} = \mathbf{X}\mathbf{V}_q$
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MC-GPCA Illustrating Examples



extract 1-4 hop graph walk statistics, reference node = node 1, $p = 9$

Structural Difference Score (SDS)

- \mathbf{Y} : an $n \times q$ matrix from MC-GPCA
- SDS of node i :

$$\text{SDS}(i) = \frac{\sum_{j \in \text{Neighbor}(i)} \|\text{row}_i(\mathbf{Y}) - \text{row}_j(\mathbf{Y})\|^2}{d_i + 1}$$

d_i : degree of node i

- Interpretation: an aggregated centrality measure of structural difference of a node and its neighboring nodes



Multi-Centrality Graph Dictionary Learning (MC-GDL)

- Given a set of g graphs. For each graph, run MC-GPCA, compute SDS, extract top z nodes of highest SDS as one column of \mathbf{Z}
- Sparse dictionary learning model: $\mathbf{Z} = \mathbf{D}\mathbf{C} + \text{noise}$. \mathbf{C} column sparse
- K-SVD [Aharon-Elad-Bruckstein, Tran. Signal Processing'06]:
$$\min_{\mathbf{D}, \mathbf{C}} \|\mathbf{Z} - \mathbf{D}\mathbf{C}\|_F^2 \quad \text{subject to } \|\mathbf{col}_j(\mathbf{C})\|_0 \leq S, \forall j$$

Algorithm 2 Multi-centrality graph dictionary learning (MC-GDL)

Input: A set of graphs $\{G_\ell\}_{\ell=1}^g$, number of atoms K , sparsity constraint S , number of highest SDS feature z

Output: graph structure dictionary \mathbf{D} , coefficient matrix \mathbf{C}

1. Obtain z highest SDS for each graph as columns of \mathbf{Z}
 2. Subtract column-wise empirical average from \mathbf{Z}
 3. Perform K-SVD on \mathbf{Z} to obtain \mathbf{D} and \mathbf{C}
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Application: Cyber Intrusion Detection

- The University of New Brunswick (UNB) intrusion detection dataset - 7 days of graph connectivity patterns of a cyber system [3]

Dataset	# nodes	# edges	Description
Day 1	5357	12887	Normal activity
Day 2	2631	5614	Normal activity
Day 3	3052	5406	Infiltrating attack and normal activity
Day 4	8221	12594	HTTP denial of service attack and normal activity
Day 5	24062	32848	Distributed denial of service attack using Botnet
Day 6	5638	13958	Normal activity
Day 7	4738	11492	Brute force SSH attack and normal activity

Extracted structural features for each graph:

- 1) 1-20 hop graph walk statistics
- 2) 6 centrality measures
- 3) 10 reference nodes of highest degree

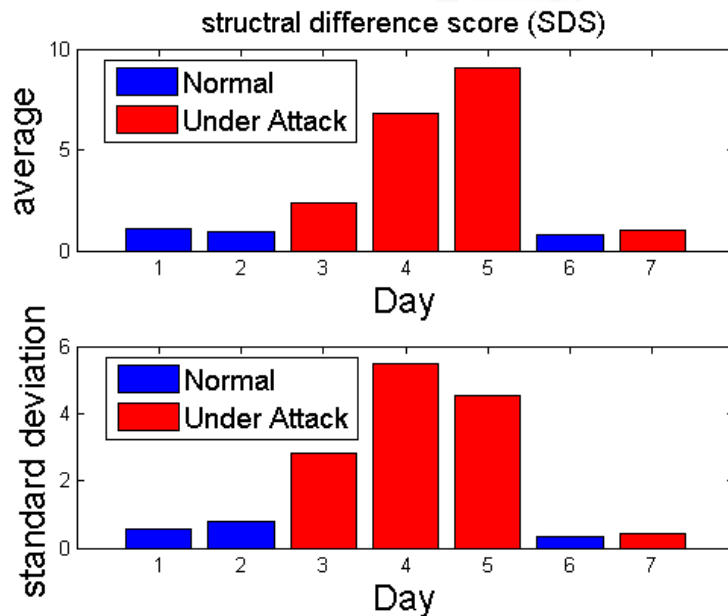
In total $p = 56$ features

Ref: Chen-Choudhury-Hero, [ICASSP'16](#)

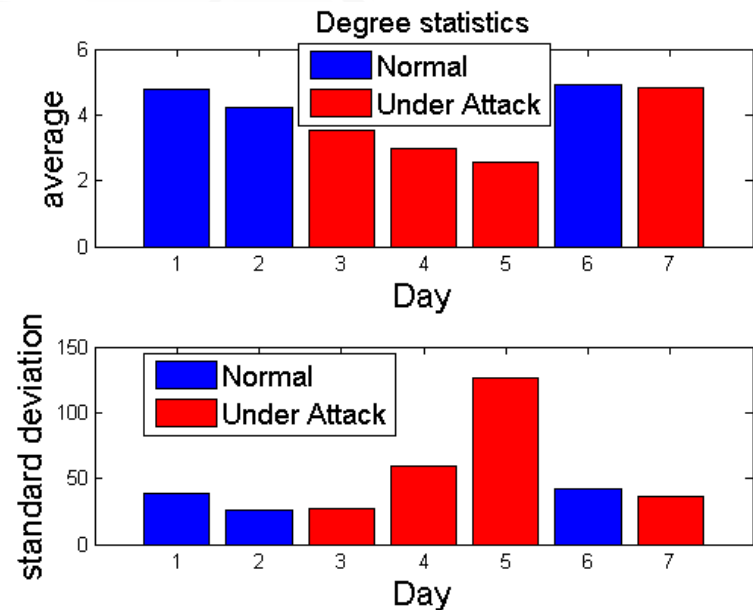
[3] Shiravi-Shiravi-Tavallaee-Ghorbani, *Computers & Security* 12

Performance Evaluation - MC-GPCA

SDS score (proposed)



Degree score



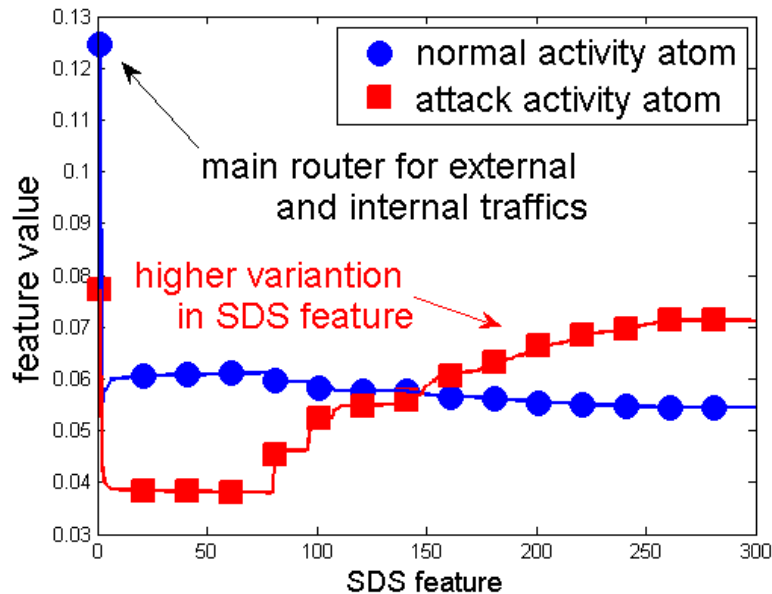
- ✓ The proposed SDS from MC-GPCA is an effective indicator of intrusions resulting in anomalous connectivity patterns (Days 3,4,5)

Ref: Chen-Choudhury-Hero, [ICASSP'16](#)

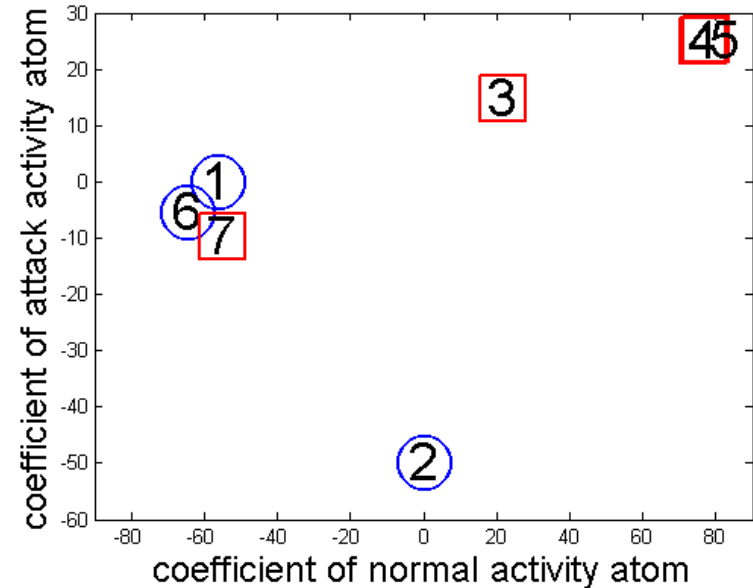


Performance Evaluation - MC-GDL

Atoms from MC-GDL



Coefficients from MC-GDL



- ✓ The atoms learned from MC-GDL reflect normal and anomalous connectivity patterns, and the coefficient matrix \mathbf{C} from MC-GDL can be used for attack classification.

Ref: Chen-Choudhury-Hero, [ICASSP'16](#)



Conclusion

- Nuclear facilities are vulnerable to cyber attacks
- Tools for early detection of cyberattacks are essential
- Proposed a multi-centrality decomposition of at-risk networks (MC-GPCA & MC-GDL)
- Proposed a structural difference score (SDS) for detecting anomalous connectivity patterns
- Demonstrated on cyber intrusion database based on pairwise connectivity of nodes



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- Contact Info: {pinyu,hero}@umich.edu

Ref: Chen-Choudhury-Hero, [ICASSP'16](#)

