Uncertainty Quantification of Fast-Neutron Multiplicity Expressions for Plutonium Mass Estimation

Michael Y. Hua*, Tony H. Shin, Angela Di Fulvio, Shaun D. Clarke, and Sara Pozzi

Department of Nuclear Engineering and Radiological Sciences, University of Michigan

*mikwa@umich.edu

Consortium for Verification Technology (CVT)

Background and Motivation

- Special nuclear material (SNM) emits time correlated neutrons and gamma rays in multiplicities. The multiplicity rates are characteristic to isotopes.
- Detected multiplicity rates (singles, doubles, and triples) can be used to analytically calculate sample parameters related to the mass, multiplication, and composition of an unknown sample; this technique is known as neutron multiplicity counting (NMC). See Refs. [1-4].

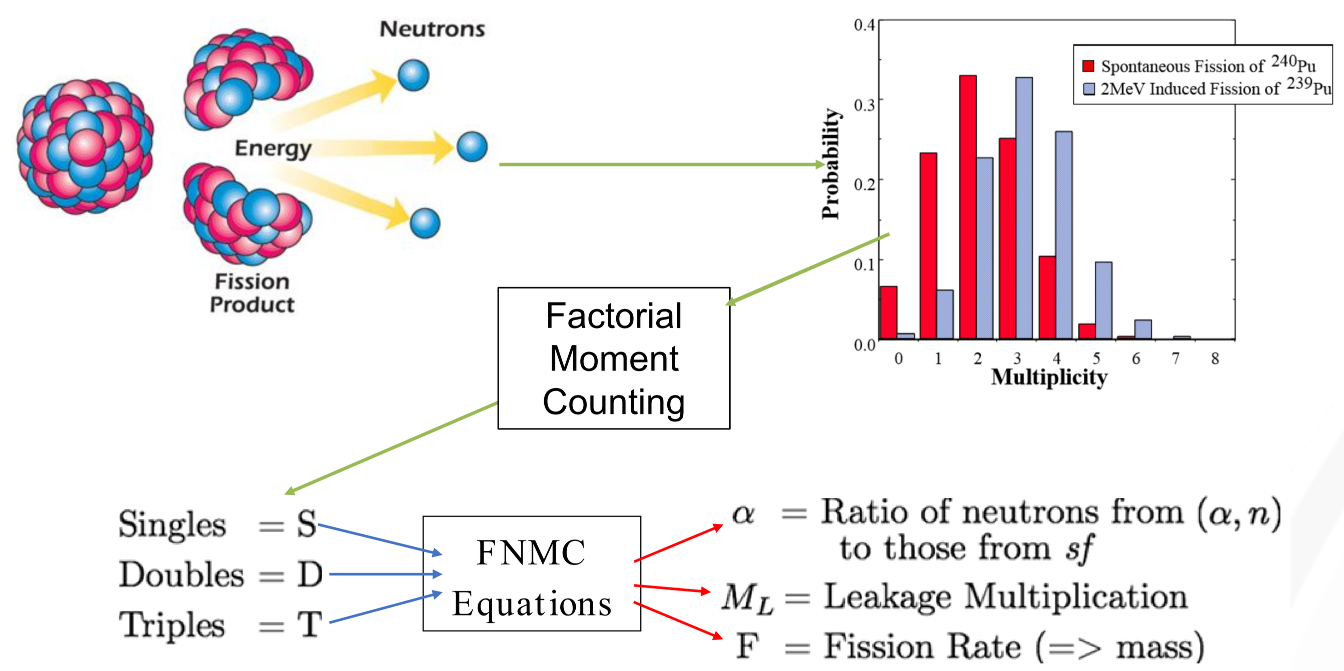

Fig 1. The emission-detection-processing path for fast-neutron multiplicity counting (FNMC).

- This uncertainty quantification (UQ) allows us to measure until a precision is met instead of measuring for a fixed amount of time.
- UQ gives a quantitative confidence interval which is needed in treaty verification, safeguards, materials management, and forensics.
- UQ can be used to perform sensitivity analysis which informs system design and parameters.
- This UQ method allows us to study the contribution of uncertainty by individual variables and it allows us to study covariances.

Theory

- For more information regarding the derivation of the analytic uncertainty equations or fast-neutron multiplicity counting, see Refs. [5] and [1] respectively.
- Sample Pulse Train:

- More definitions:
  - $B_x(\tau) :=$ number of windows τ with exactly x signals after K random-triggered windows
  - $N_x(\tau) :=$ number of windows τ with exactly x signals after N_T signal-triggered windows

- $m_b(\mu) \approx \sum_{x=\mu}^{\infty} \binom{x}{\mu} \frac{B_x(\tau)}{K}$ Reduced factorial moment from random triggering
- $m_n(\mu) \approx \sum_{x=\mu}^{\infty} \binom{x}{\mu} \frac{N_x(\tau)}{N_T}$ Reduced factorial moment from signal triggering

- $S = \frac{m_{n(0)}}{\tau}$
- $D = \frac{m_{n(1)} - m_{b(1)}}{\tau}$
- $T = \frac{m_{n(2)} - 2m_{b(1)}m_{n(1)} - \frac{5}{2}m_{b(1)}^2}{\tau}$

- $F :=$ Fission rate $= F(S, D, T)$
- $c = \left[\frac{fissions}{s \cdot g}\right]$ = isotope-characteristic constant
- $m :=$ Sample effective mass = $\left[\frac{F}{c}\right]$

$$\boxed{c^2\sigma_m^2 = \sigma_F^2 = \left(\frac{\partial F}{\partial S}\right)^2 \sigma_S^2 + \left(\frac{\partial F}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial F}{\partial T}\right)^2 \sigma_T^2 + 2\left(\frac{\partial F}{\partial S}\frac{\partial F}{\partial D}\right)\sigma_{SD} + 2\left(\frac{\partial F}{\partial S}\frac{\partial F}{\partial T}\right)\sigma_{ST} + 2\left(\frac{\partial F}{\partial D}\frac{\partial F}{\partial T}\right)\sigma_{DT}}$$

Experiment

The FNMC system was used to measure a mixed plutonium source. The plutonium was in the form of metal plates (with 4.72 g ^{240}Pu effective mass per plate) and the sample mass was varied by varying the number of plates (1, 3, 5, or 7). The collection time was varied between 1 and 30 minutes. The experiment geometry and setup can be seen in Figure 2.

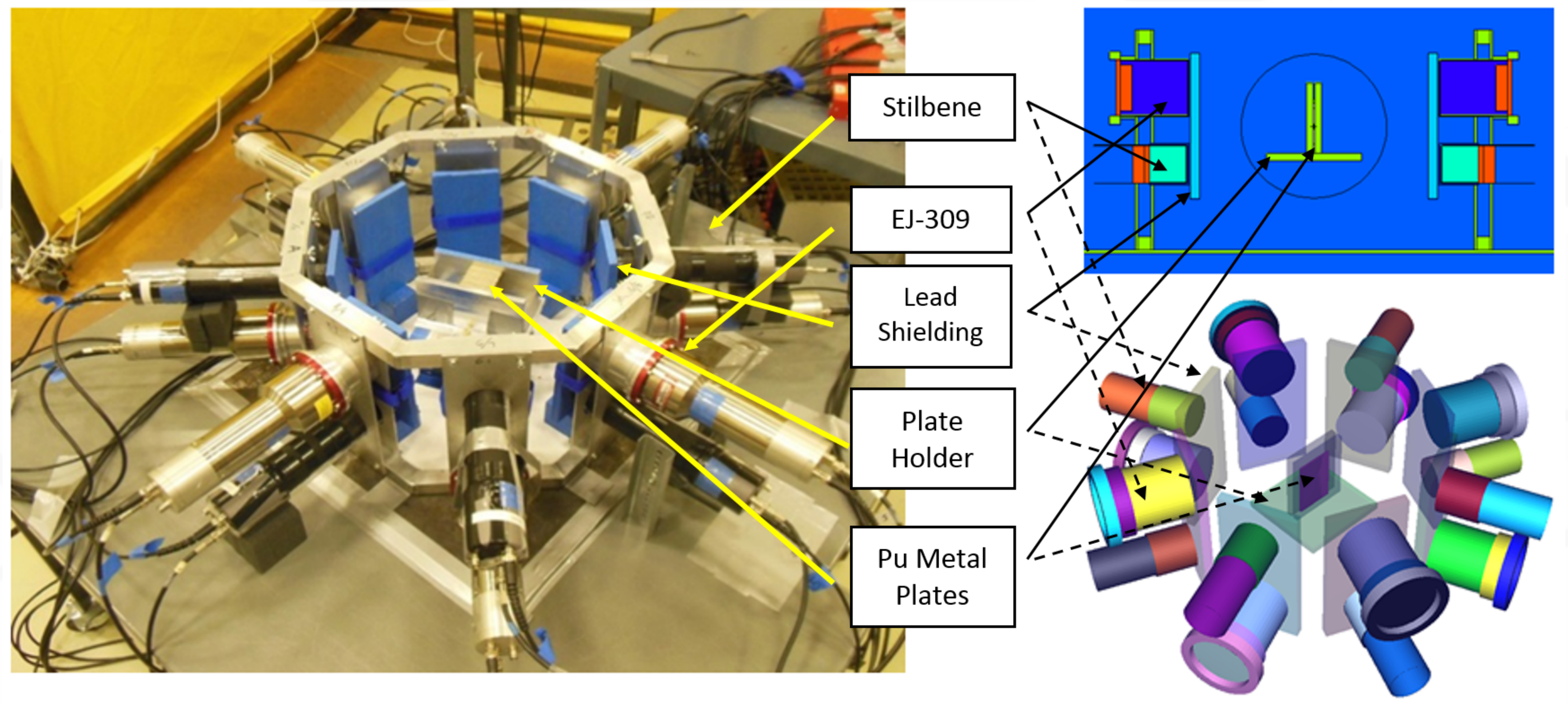

Fig 2. (Left) photograph of the experimental setup of the FNMC system. (Right) the MCNPX-PoliMi simulation model used to determine the detection efficiencies.

Results

The detected neutron multiplicity distribution was processed with factorial moment counting to determine the multiplicity rates. Next, the FNMC equations were used to calculate the sample effective mass. Then, the uncertainty (one standard deviation) was calculated in three ways: (1) by using the analytic equations that account for covariance Ref. [5], (2) by using the same equations as case (1) but omitting the covariance terms, and (3) by assuming S, D, and T are independent, Poisson random variables Ref. [6]. The results are tabulated in Table 1 and are depicted in Figure 3.

*Table 1. The 1-σ **relative** uncertainty in the estimated effective mass, calculated including and excluding the covariance, and by assuming S, D, and T are Poisson random variables.*

^{240}Pu True Effective Mass (g)	Measurement Time (min)	Estimated Effective Mass (g)	Covariance Included	Covariance Excluded	Poisson Assumption
4.72	1	4.251	0.232	0.271	0.265
	2	4.044	0.168	0.197	0.195
	5	4.378	0.098	0.113	0.115
	10	4.242	0.072	0.084	0.084
	20	4.463	0.050	0.058	0.057
	30	4.421	0.041	0.048	0.047
14.16	10	13.898	0.042	0.050	0.043
23.60	10	29.079	0.024	0.029	0.026
33.04	10	39.142	0.018	0.022	0.022

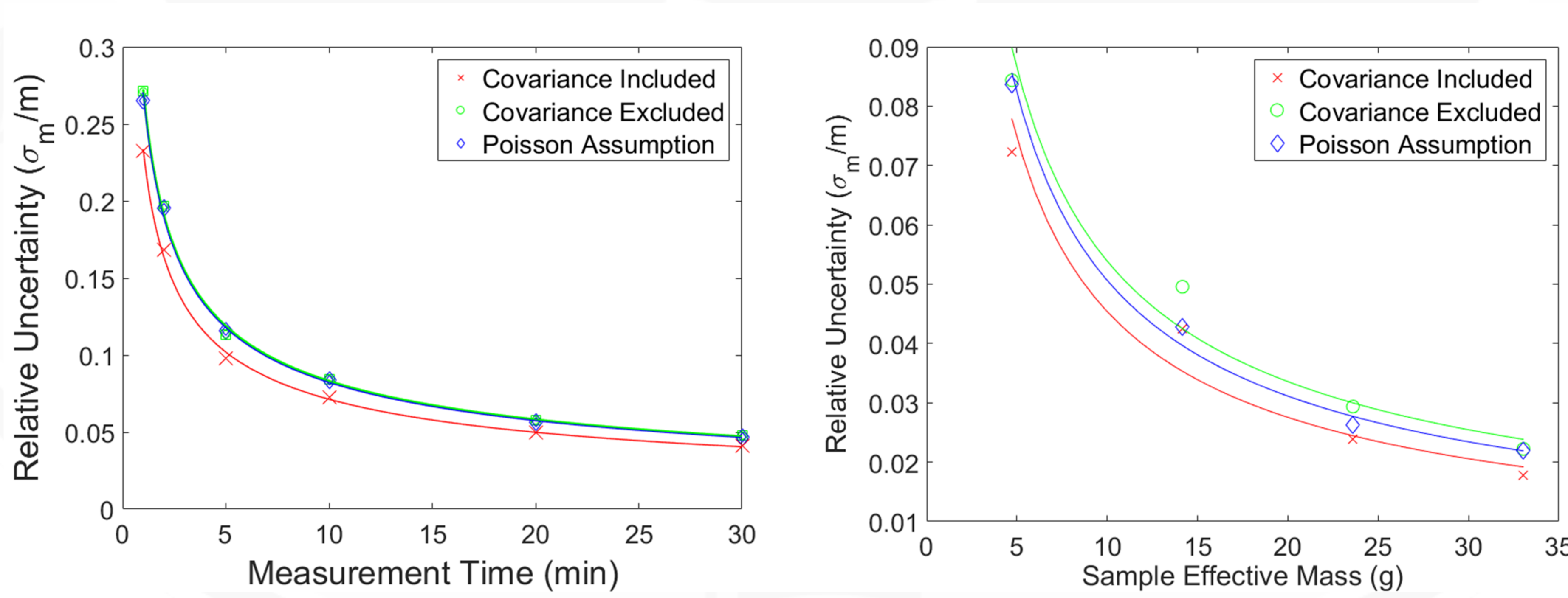

Fig 3. (Left) plot of the relative uncertainty as a function of total measurement time for a single plutonium plate (4.72 g). (Right) plot of the relative uncertainty as a function of the ^{240}Pu effective mass for a total measurement time of 10 minutes. The power-fits vary as $1/\sqrt{x}$.

Discussion of Results

- The relative uncertainty decreases as a function of increasing measurement time/sample effective mass for fixed sample effective mass/measurement time…the decrease varies as $x^{-\frac{1}{2}}$, just as Monte Carlo convergence.
- In general, the uncertainties calculated under the Poisson assumption deviate from the uncertainty calculated with the analytic equations omitting the covariance terms (assuming independence) by less than 3%.
- As seen in Figure 3, including the covariance terms reduces the relative uncertainty.
- As seen in Figure 4, an optimal measurement time can be pre-determined when given a refinement criterion. For example, if the criterion for increasing the total measurement time by one minute is a reduction in the relative uncertainty by 7%, a one-minute measurement time is sufficient (since increasing the measurement time to two minutes only reduces the relative uncertainty by 6.4%).

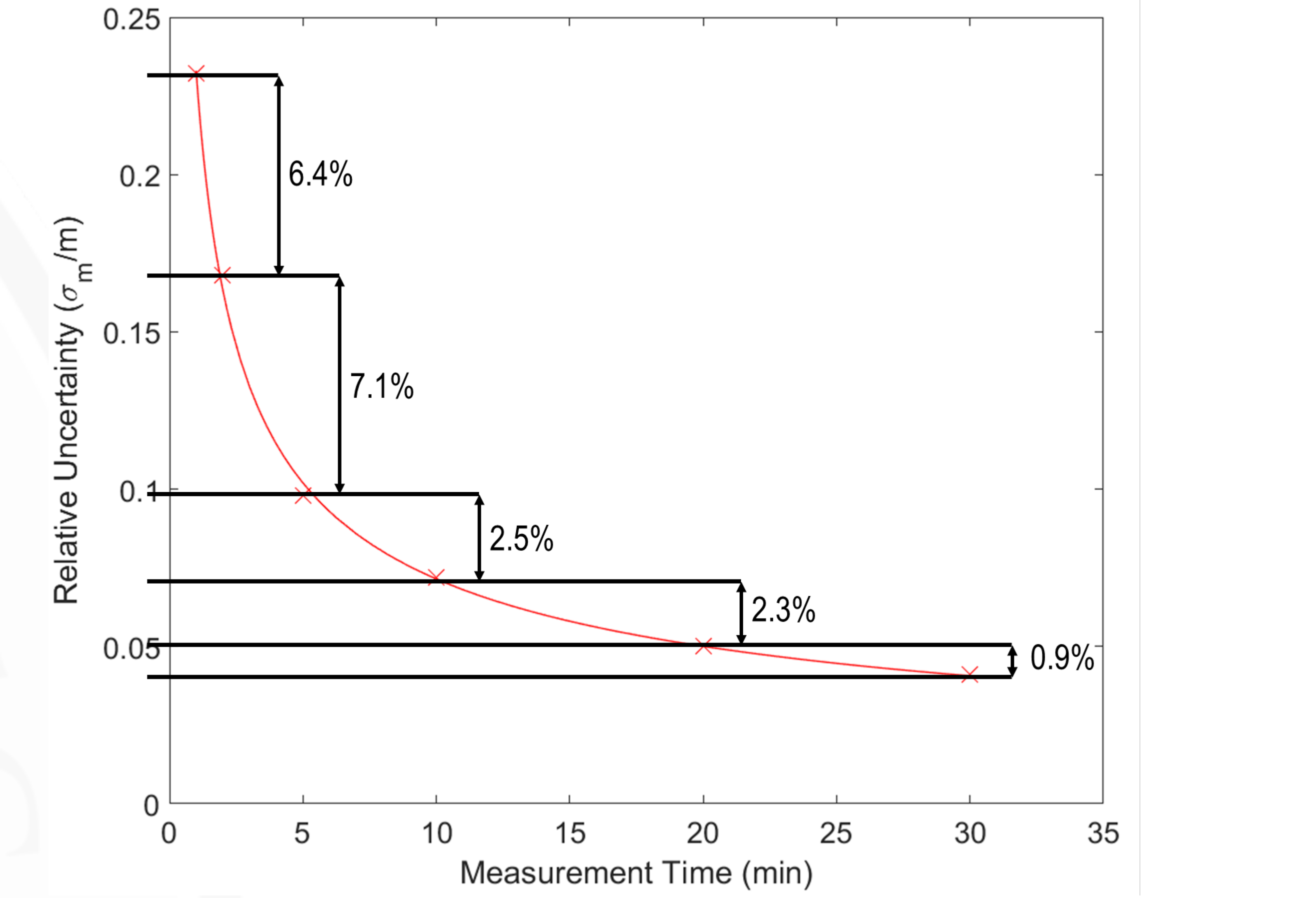

Fig 4. An annotated plot of relative uncertainty vs. measurement time, showing the reduction in relative uncertainty between various total measurement times.

Conclusions

- Given a refinement criterion, an optimal measurement time can be determined. Furthermore, a measurement can last until a certain precision is met as opposed to running for an arbitrary amount of time.
- Inclusion of the covariance terms results in a more accurate uncertainty that is less than the estimated uncertainty when the covariance terms are omitted. This means that the desired precision is truly reached earlier, shortening the total measurement time, and therein reducing procedural and operational costs.
- Neutron multiplicity rates are not independent, Poisson random variables; they inherently covary and are dependent on one another.

References

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