An Efficient Nonlinear Dispersion Model Using Continuous-Piecewise-Affine-based Transformations

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Scenario

Plume

Wind Field

Emission Source

Airborne Sensor
Motivation

Goal: detect an *unknown number* of emitters of material (e.g. radiological) via airborne sensors.

Challenges:
1. Airborne material subject to complex transport and dispersion effects
2. Number of sources is unknown
Related Work

- Single source
- Low ambient concentrations
- Small survey area

- Known number of sources
- Low ambient concentrations
- Small survey area

Johannesson et al. (2004); Keats et al. (2007); Many others

- Unknown number of sources
- Time-varying emissions
- Low ambient concentrations
- Small survey area

- Unknown number of sources
- High ambient concentrations
- Very large survey area

Yee (2008); Yee (2007); Hirst et al. (2013)
Our Contributions

- Efficient nonlinear propagation modeling using CPAB flow models
- Multi-flight models for efficient inference
- Uncertainty quantification allows for extension to information planning

(Freifeld et al., 2015)
Probabilistic Graphical Model

Number of emission sources
(Unknown!)

$K$

$z_k$

$w_k$

$s_k$

$\eta_z$

$\eta_w$

$\eta_s$
Probabilistic Graphical Model

Number of emission sources
(Unknown!)

Number of measurements

\[ K \]
\[ N \]
\[ U \]

\[ \eta_z \]
\[ \eta_w \]
\[ \eta_s \]
\[ \eta_\sigma \]
Probabilistic Graphical Model

Number of emission sources
(Unknown!)

Number of measurements

\( K \)

\( z_k \)

\( \eta_z \)

\( w_k \)

\( \eta_w \)

\( s_k \)

\( \eta_s \)

\( N \)

\( t_i \)

\( x_i \)

\( \eta_b \)

\( U \)

\( b \)

\( \eta_\sigma \)
Probabilistic Graphical Model

Number of emission sources
(Unknown!)

Number of measurements in flight \( j \)

Number of flights

\( \eta_z \), \( \eta_w \), \( \eta_s \), \( \eta_b \), \( \eta_\sigma \)
Observation Model

\[ y_{ji} \sim \mathcal{N} \left( \sum_k a_{ijk} s_{kj} + b_{ji}, \sigma_j^2 \right) \]

\[ \mathcal{N}(b_j; \mu_b, \Lambda_j^{-1}) \]

**Background Model**

\[ a_{ijk} \triangleq A(t_{ji}, x_{ji}, z_k, w_k; U_j) \]

**Atmospheric Dispersion Model**

\[ \sigma_j^2 \sim \text{IG}(\eta, \sigma) \]

**Conjugate Prior on Measurement Error**

**Coupling Coefficients**
Modelling Particle Transport

- Velocity fields (e.g. wind) can be represented as **diffeomorphisms**: transformations that are both smooth and invertible.

- Simulating particle trajectories for an arbitrary vector field is computationally prohibitive for large-scale inference.
CPAB Particle Transport

• Continuous-piecewise affine (CPA) velocity fields are a flexible class of diffeomorphism that lead to highly-efficient trajectory simulation.

• CPA-based transformations are piecewise w.r.t. a tessellation of the space that limits field complexity.
Atmospheric Dispersion Model
Gaussian Plume Model

\[ \sigma_H = \sqrt{(\delta_R \sigma_\theta)^2 + \omega^2} \]

\[ \sigma_V = \delta_R \sigma_\phi \]
Nonlinear Gaussian Plume Model

Warp the plume center-line using CPAB trajectory simulation!
Background Model

Physical Process:

Gaussian Markov Random Field:

\[ p(b_j; \eta_b) = \mathcal{N}(b_j; \mu_b, \Lambda_j^{-1}) \]

Mean Vector

Precision Matrix

(Uses CPAB Simulation!)
Contribution Summary

• Propagation modeling using CPAB flow models
  – Dispersion modeling
  – Background modeling
• Multi-flight models for efficient inference
Experiment: Real Data
Posterior Analysis

Estimate any arbitrary function $f(\theta)$ of the posterior $p(\theta \mid y)$, where $\theta$ represents the set of latent variables using Monte Carlo integration.

$$\mathbb{E}_{\theta \mid y} [f(\theta) \mid y] \approx \frac{1}{T} \sum_{t=1}^{T} f(\theta^{(t)}) \quad \text{where} \quad \theta^{(t)} \sim p(\theta \mid y)$$

e.g. Expected Probability of Event A
(i.e. Marginal Distributions): $f(\theta; A) = \mathbb{I}(\theta \in A)$

Expected Emission Rate in Area $\mathcal{A}$:

$$f(\theta; \mathcal{A}) = \sum_{k} s_{kj} \int_{\mathcal{A}} \mathcal{N}(z; z_k, \mathbf{I}w_k^2) \, dz$$
Expected Emission Rate

Expected Source Emissions Originating in Area $\mathcal{A}$:

$$f(\theta; \mathcal{A}) = \sum_{k} s_{k,j} \int_{\mathcal{A}} \mathcal{N}(z; z_k, Iw_k^2) \, dz$$
Synthetic Experiment: Linear vs. CPAB Wind Model

- Simulate observations using CPAB wind model.
- Compare inference using linear vs CPAB model.
- E.g. shift by 10°/1000 m.
Synthetic Experiment: Angular Shift: 0° / km

Actual Flow Field

Linear Wind Model

CPAB Wind Model
Synthetic Experiment: Angular Shift: 2.5° / km

Actual Flow Field  Linear Wind Model  CPAB Wind Model
Synthetic Experiment: Angular Shift: 5° / km

Actual Flow Field  Linear Wind Model  CPAB Wind Model

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Synthetic Experiment:
Angular Shift: 10° / km

Actual Flow Field  |  Linear Wind Model  |  CPAB Wind Model

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Single Slides
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Synthetic Experiment: Angular Shift: 20° / km

Actual Flow Field  Linear Wind Model  CPAB Wind Model

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Summary

• The CPAB wind model measurably improves localization accuracy over existing models.
• Sampling-based inference leads to efficient methods of evaluating system performance.
• We have demonstrated only a small subset of possible reasoning tasks over the posterior.
• This is a general purpose model that is applicable to many different material detection applications.
Comments & Questions
Bonus Slides
Model Description
Model of Emissions

We observe *continuous* releases from an unknown number of emission sources.

Each emission source is described by:

**Emission Location**

Normally-distributed with **mean** and isotropic **covariance** matrix

\[ N(z, I w^2) \]

**Emission Rate**

(assumed **constant**)

\[ \text{time (sec)} \]
Each flight described by:
- Concentrations $y$ at times $t$ and locations $x$
- Wind field $U$
- Measurement error variance $\sigma^2$
Ambient Substance Levels

- Our sensor captures both source emissions and spatio-temporally smooth **background concentrations**

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>t = 0 min</td>
</tr>
<tr>
<td>10</td>
<td>t = 10 min</td>
</tr>
<tr>
<td>30</td>
<td>t = 30 min</td>
</tr>
</tbody>
</table>

...
Gaussian Markov Random Fields

\[
\{i, j\} \in \mathcal{E} \\
\Lambda_{ij} = \Lambda_{ji} = -\alpha_{ij} \neq 0
\]

\[
\alpha_{ij} = \frac{1}{(c_T |\Delta T_{ij}| + c_D \Delta D_{ij})^2}
\]

\[
\Lambda_{ii} = -\sum_{j:j \sim i} \Lambda_{ij}
\]
Multi-flight Surveys

We model emission rates and background fields as *virtually independent* from flight-to-flight.
Inference
Posterior Distribution

\[ p(z, w, s, b, \sigma \mid y, t, x, U) \propto \prod_{k=1}^{K} p(z_k; \eta_z) p(w_k; \eta_w) \prod_{j=1}^{J} p(s_{kj}; \eta_s)p(b_j; \eta_b) p(\sigma_j; \eta_\sigma) \]

\[ \times \prod_{j=1}^{J} \prod_{i=1}^{N_j} p(y_{ji} \mid z, w, s_j, b_{ji}, \sigma_j, t_{ji}, x_{ji}, U_j) \]

**Posterior \propto Prior \times Likelihood**
Why is Inference Hard?

• Unknown number of sources!
• Transport is uncertain
• Large number of latent variables
• Lack of closed-form analytical expressions
• Low signal-to-noise ratio
Inference Procedure

1. Obtain initial values via optimization
2. Use RJ-MCMC to draw samples of the posterior distribution
3. Use samples from the posterior distribution for analysis
Reversible-Jump MCMC

Birth:

Death:

Split:

Merge: