An Efficient Nonlinear Dispersion Model Using Continuous-Piecewise-Affine-based Transformations

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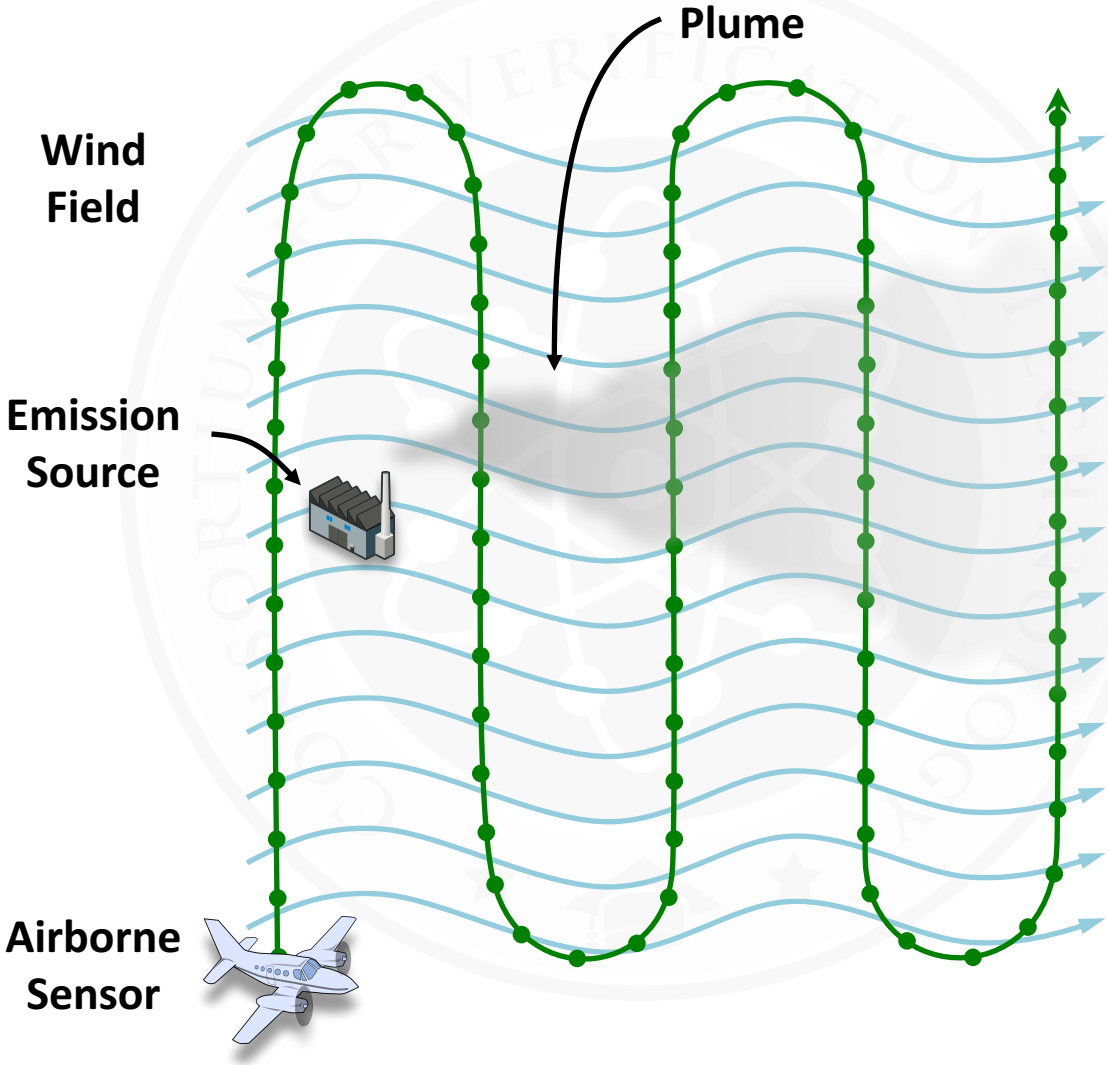
Scenario



Motivation

Goal: detect an *unknown number* of emitters of material (e.g. radiological) via airborne sensors.

Challenges:

1. Airborne material subject to complex transport and dispersion effects
2. Number of sources is unknown

Related Work

- Single source
- Low ambient concentrations
- Small survey area

Johannesson et al. (2004); Many others

- Known number of sources
- Low ambient concentrations
- Small survey area

Keats et al. (2007); Yee (2007)

- Unknown number of sources
- Time-varying emissions
- Low ambient concentrations
- Small survey area

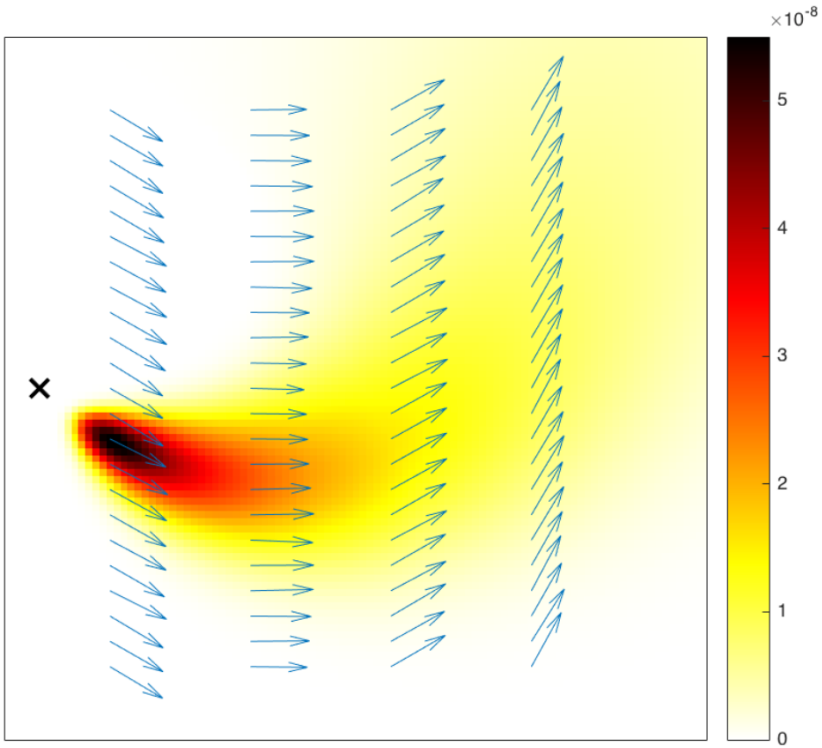
Yee (2008)

- Unknown number of sources
- High ambient concentrations
- Very large survey area

Hirst et al. (2013)

Our Contributions

- Efficient nonlinear propagation modeling using CPAB flow models
- Multi-flight models for efficient inference
- Uncertainty quantification allows for extension to information planning

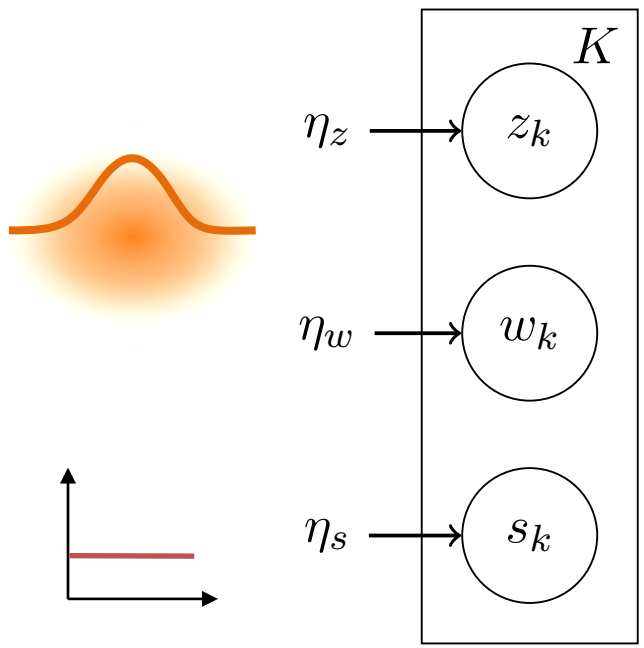


(Freifeld et al., 2015)

Probabilistic Graphical Model

Number of emission sources

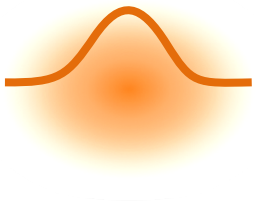
(Unknown!)

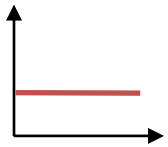


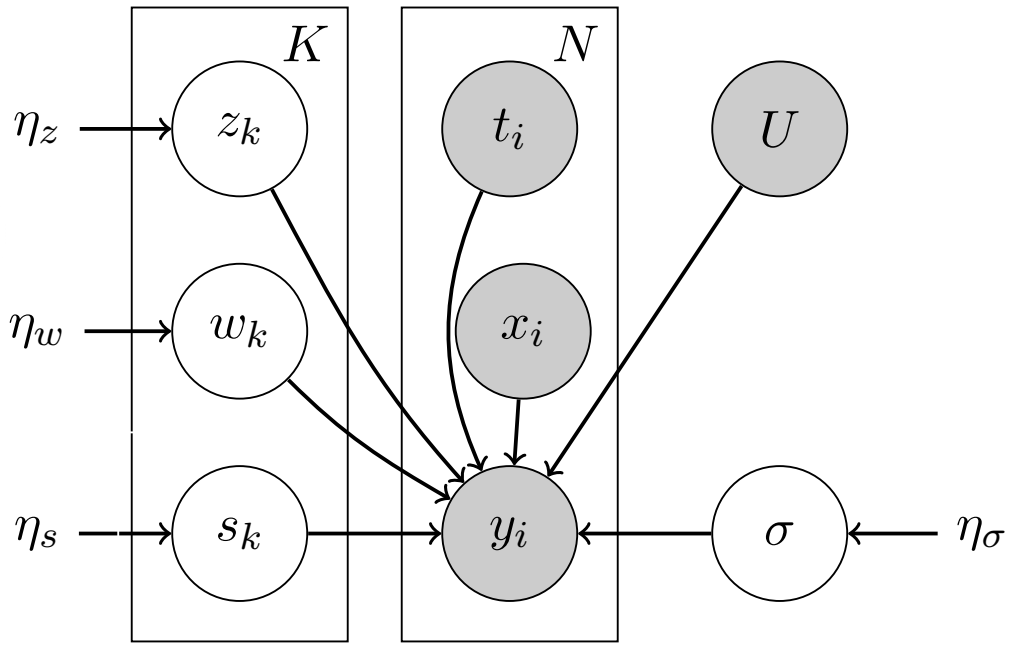
Probabilistic Graphical Model

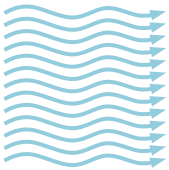
Number of emission sources
(Unknown!)

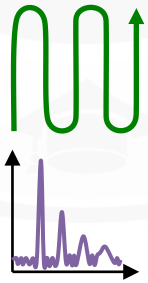
Number of measurements







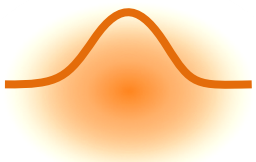


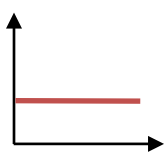


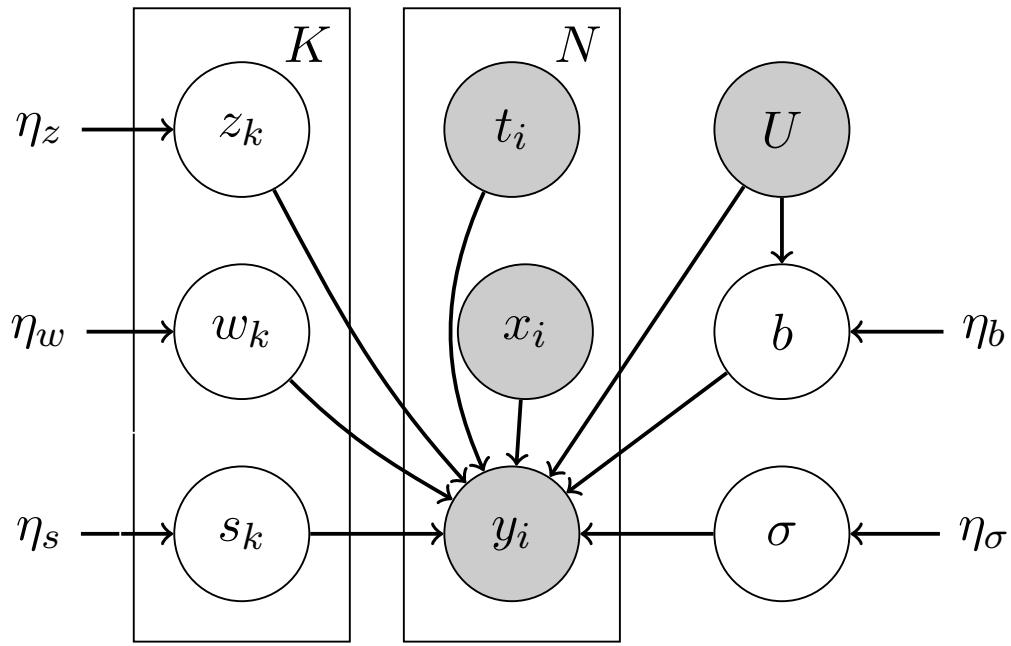
Probabilistic Graphical Model

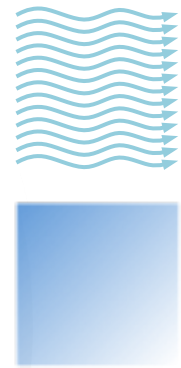
Number of emission sources
(Unknown!)

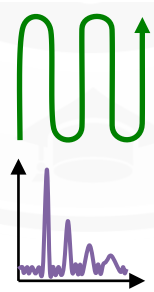
Number of measurements









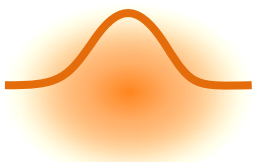


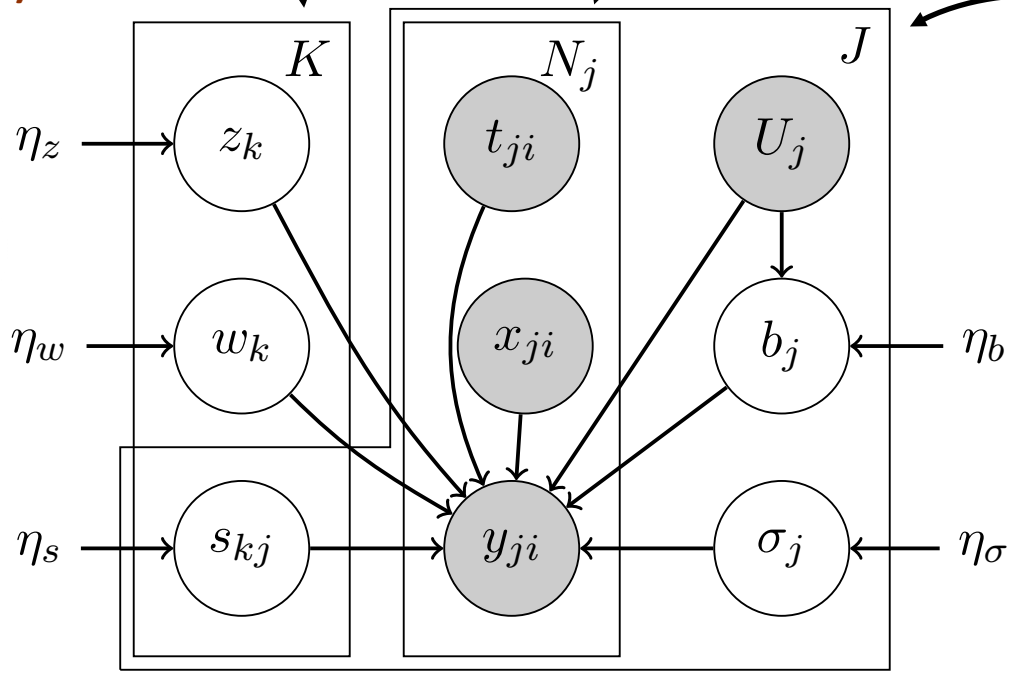
Probabilistic Graphical Model

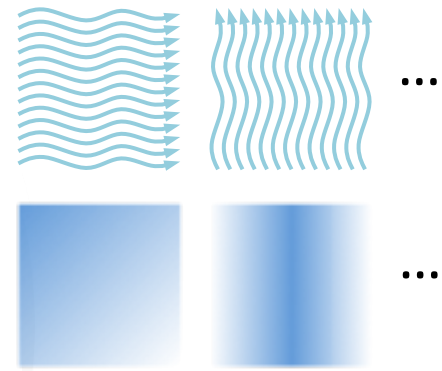
Number of emission sources
(Unknown!)

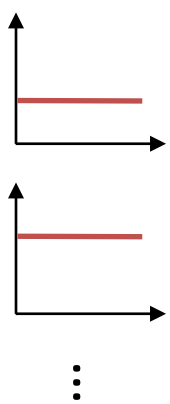
Number of measurements in flight *j*

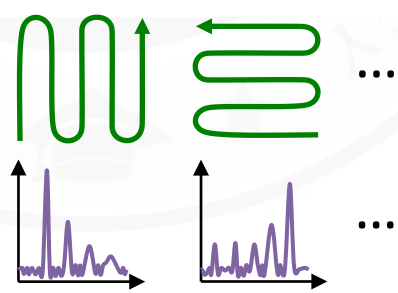
Number of flights











Observation Model

$$y_{ji} \sim \mathcal{N}\left(\sum_k a_{ijk}\, s_{kj} + b_{ji},\ \sigma_j^2\right)$$

$$\mathcal{N}(b_j;\ \mu_b, \Lambda_j^{-1})$$

Background Model

$$a_{ijk} \triangleq A(t_{ji}, x_{ji}, z_k, w_k;\ U_j)$$

$$\sigma_j^2 \sim \mathrm{IG}(\eta_\sigma)$$

Coupling Coefficients

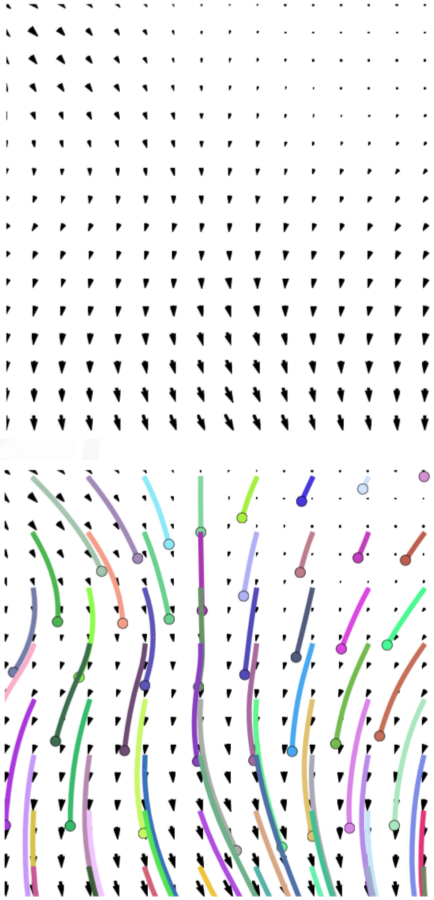
Atmospheric Dispersion Model

Conjugate Prior on Measurement Error

Modelling Particle Transport

- Velocity fields (e.g. wind) can be represented as *diffeomorphisms*: transformations that are both smooth and invertible.

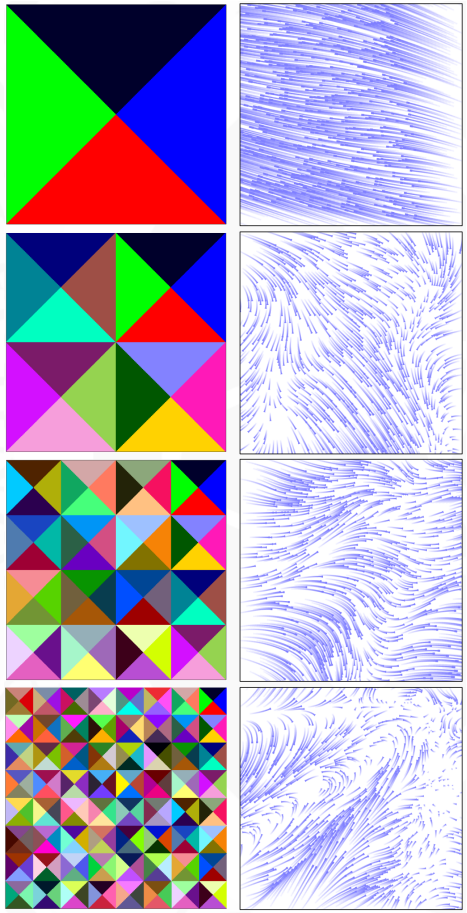
- Simulating particle trajectories for an arbitrary vector field is computationally prohibitive for large-scale inference.



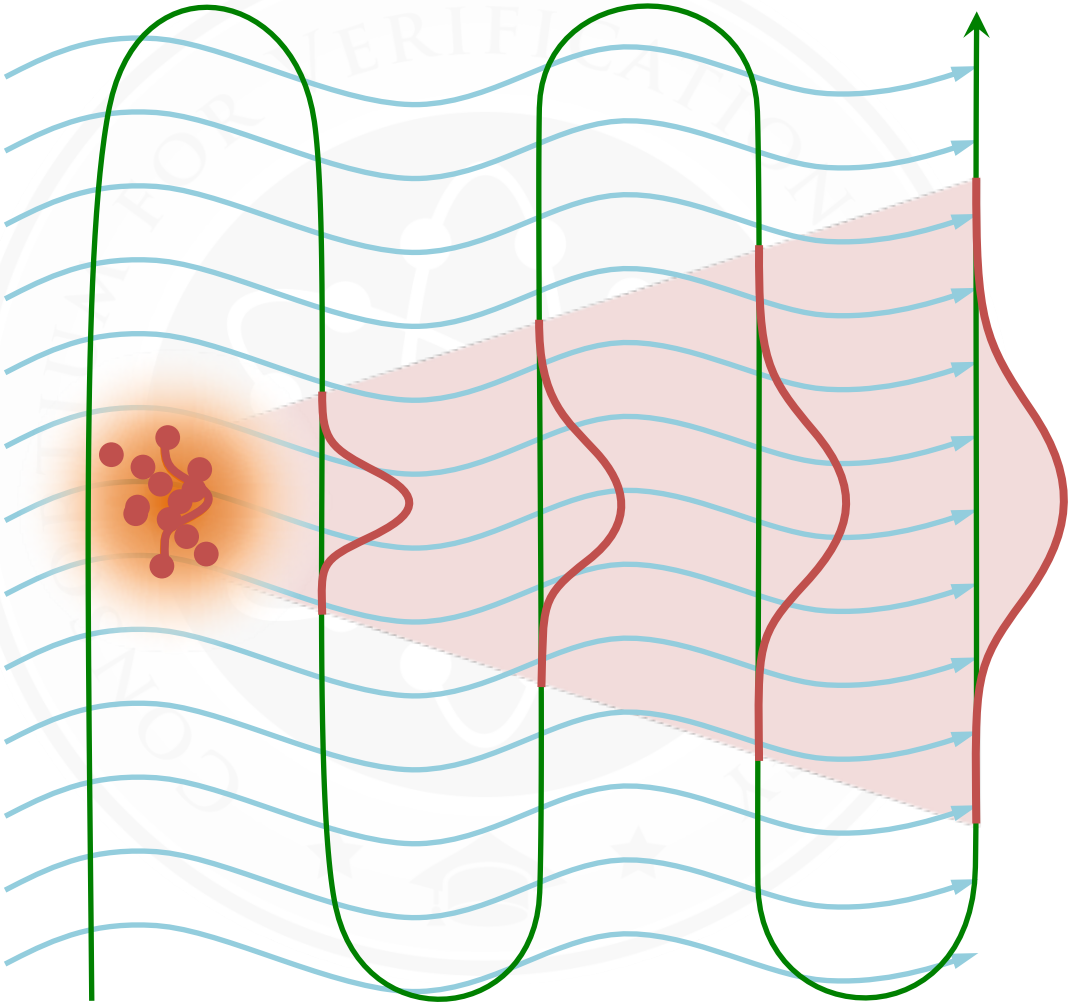
CPAB Particle Transport

- Continuous-piecewise affine (CPA) velocity fields are a flexible class of diffeomorphism that lead to highly-efficient trajectory simulation.

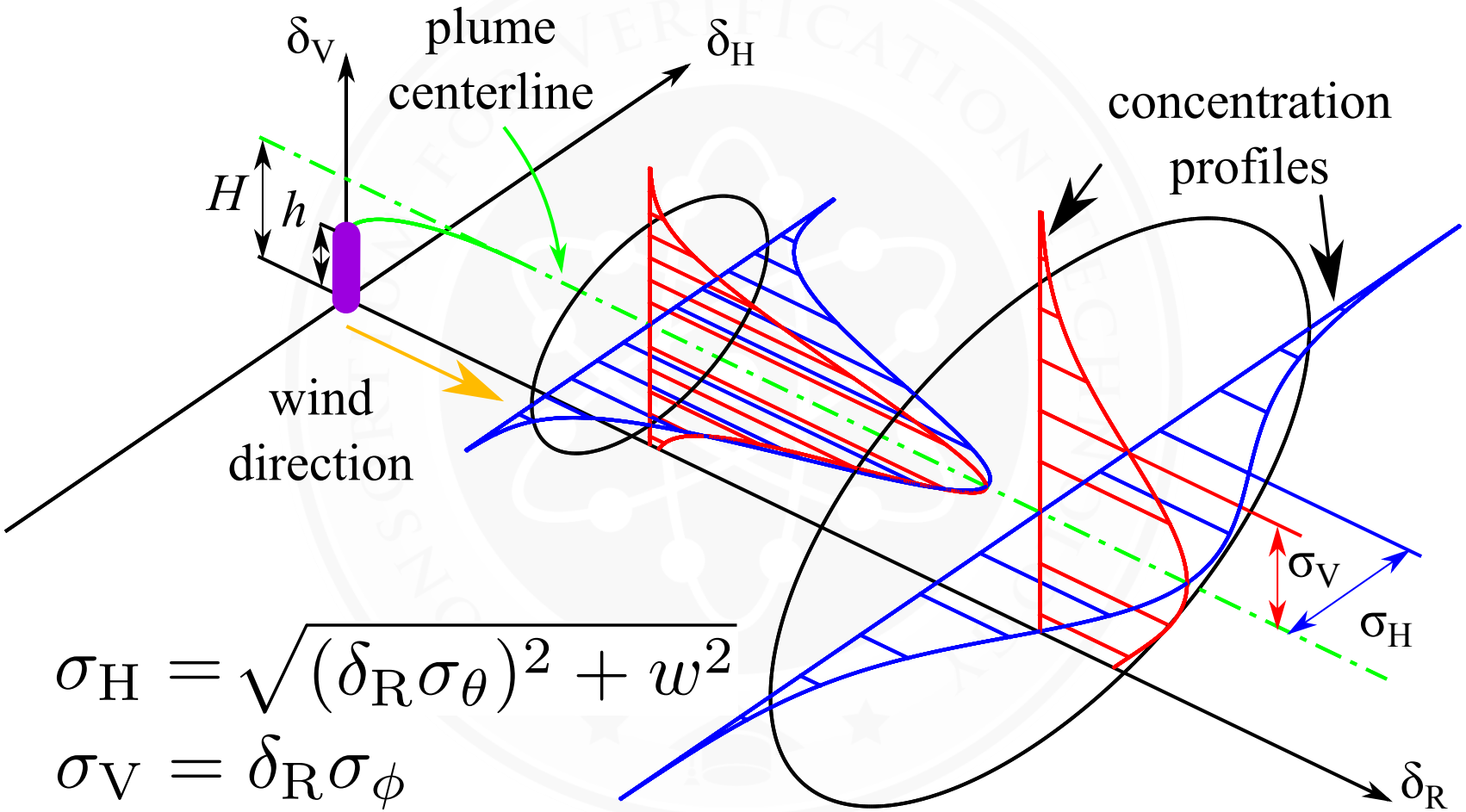
- CPA-based transformations are *piecewise* w.r.t. a tessellation of the space that limits field complexity.



Atmospheric Dispersion Model



Gaussian Plume Model

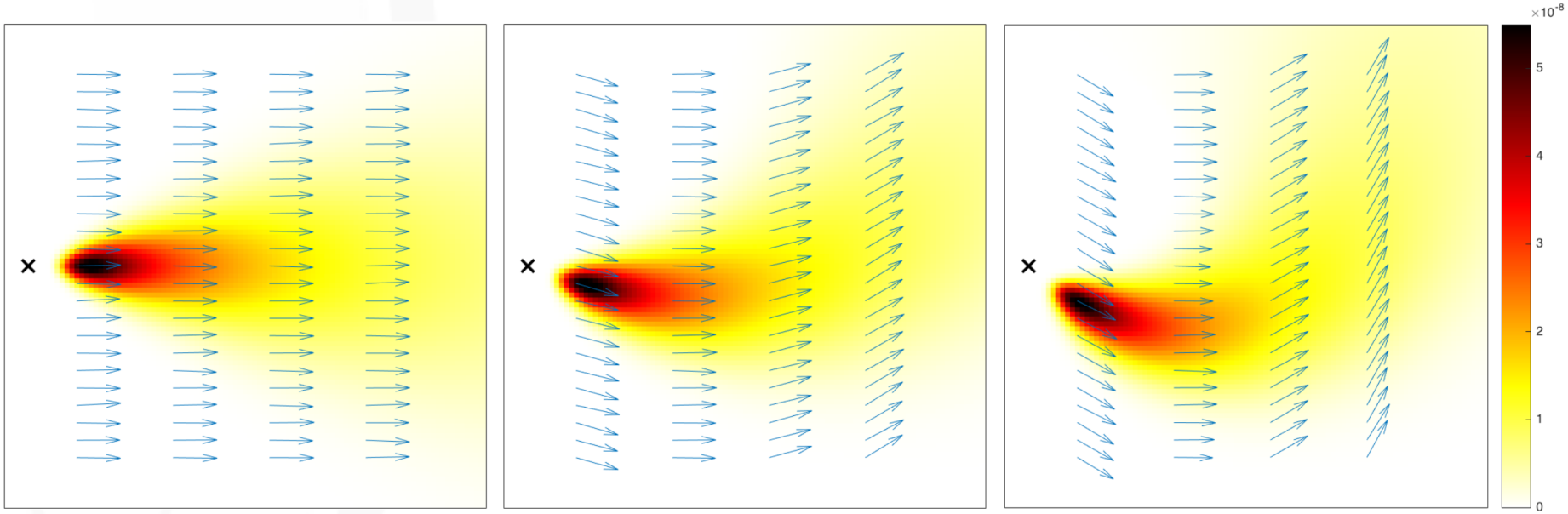


$$\sigma_{\mathrm{H}} = \sqrt{(\delta_{\mathrm{R}}\sigma_{\theta})^2 + w^2}$$

$$\sigma_{\mathrm{V}} = \delta_{\mathrm{R}}\sigma_{\phi}$$

Nonlinear Gaussian Plume Model

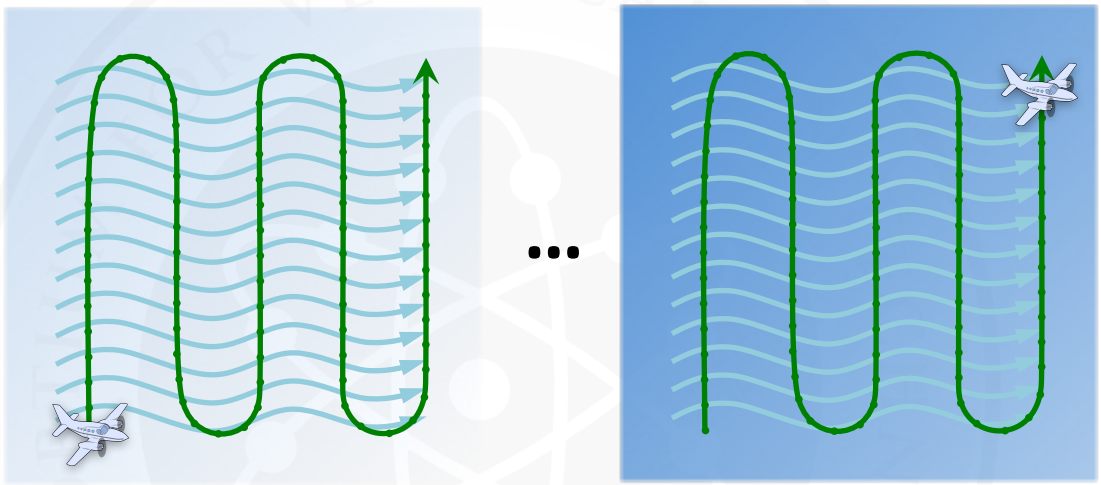
Warp the plume center-line using
CPAB trajectory simulation!



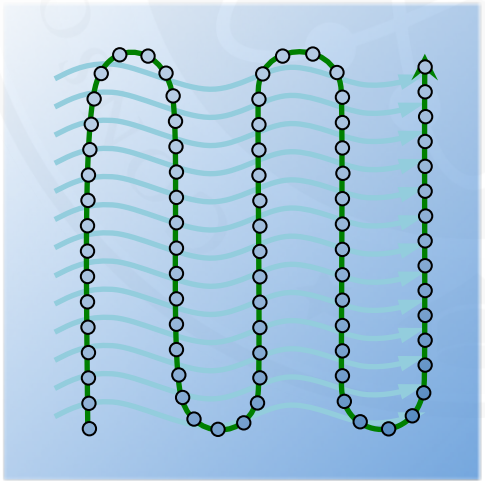
Background Model

t = 0 min **t = 30 min**

Physical Process:



Gaussian Markov Random Field:



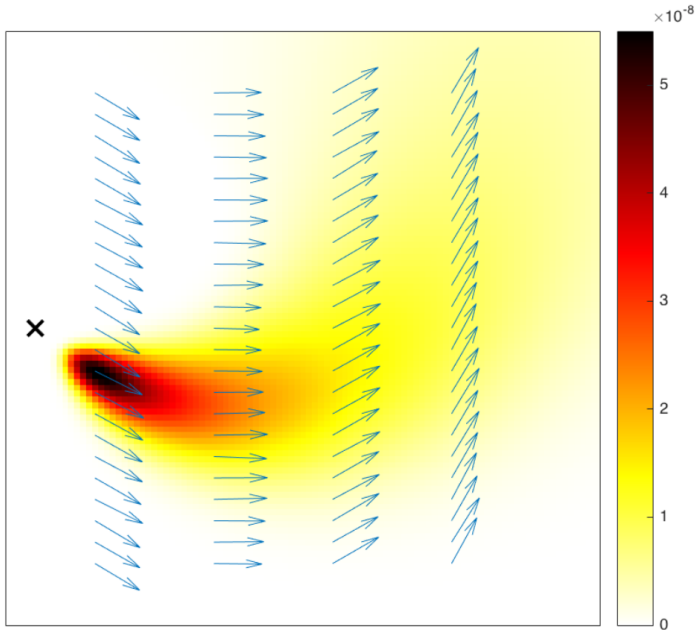
$$p(b_j;\ \eta_b) = \mathcal{N}(b_j;\ \mu_b, \Lambda_j^{-1})$$

Mean Vector *Precision Matrix*

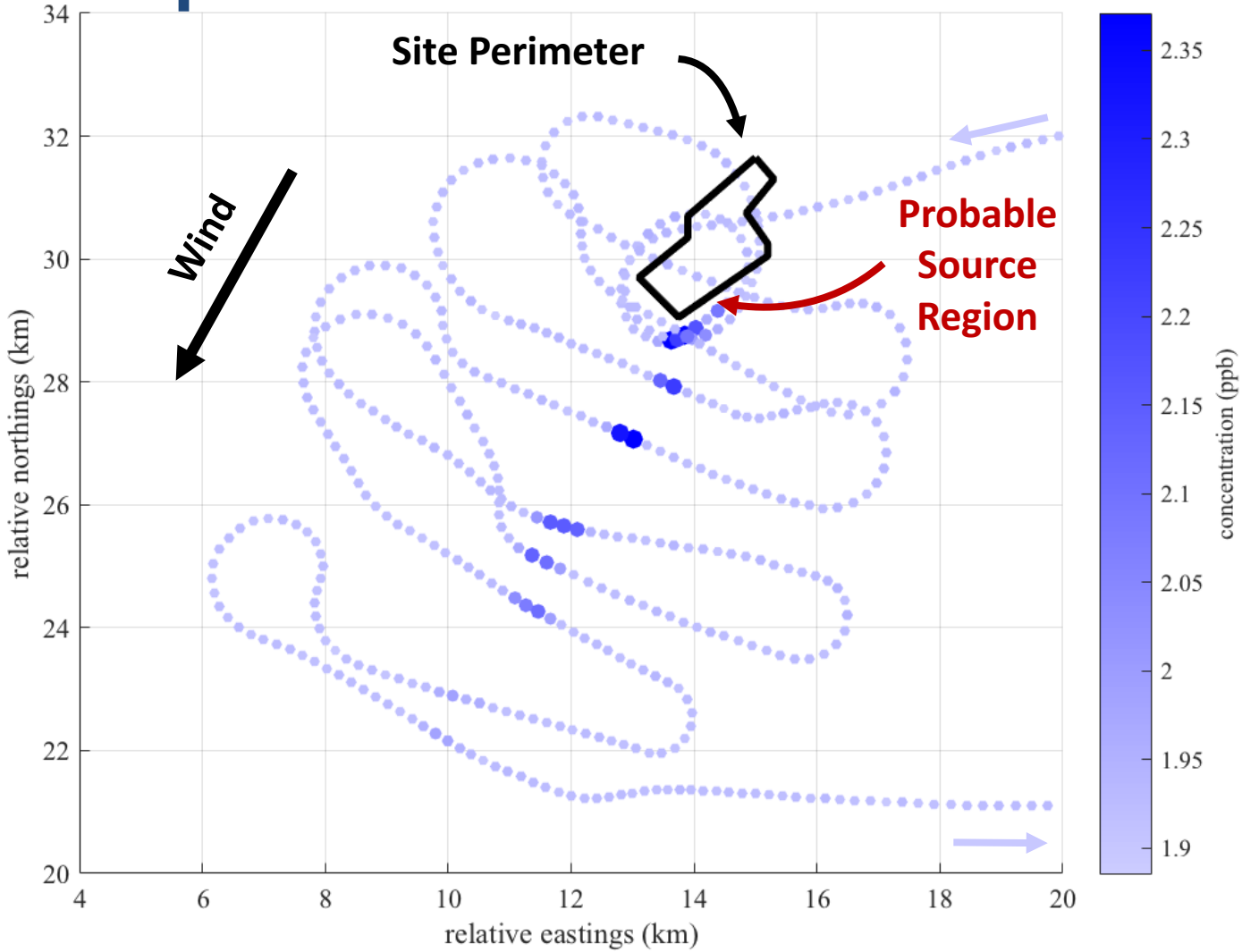
(Uses CPAB Simulation!)

Contribution Summary

- Propagation modeling using CPAB flow models
  - Dispersion modeling
  - Background modeling
- Multi-flight models for efficient inference



Experiment: Real Data



Posterior Analysis

Estimate any arbitrary function $f(\boldsymbol{\theta})$ of the posterior $p(\boldsymbol{\theta} \mid \boldsymbol{y})$, where $\boldsymbol{\theta}$ represents the set of latent variables using Monte Carlo integration.

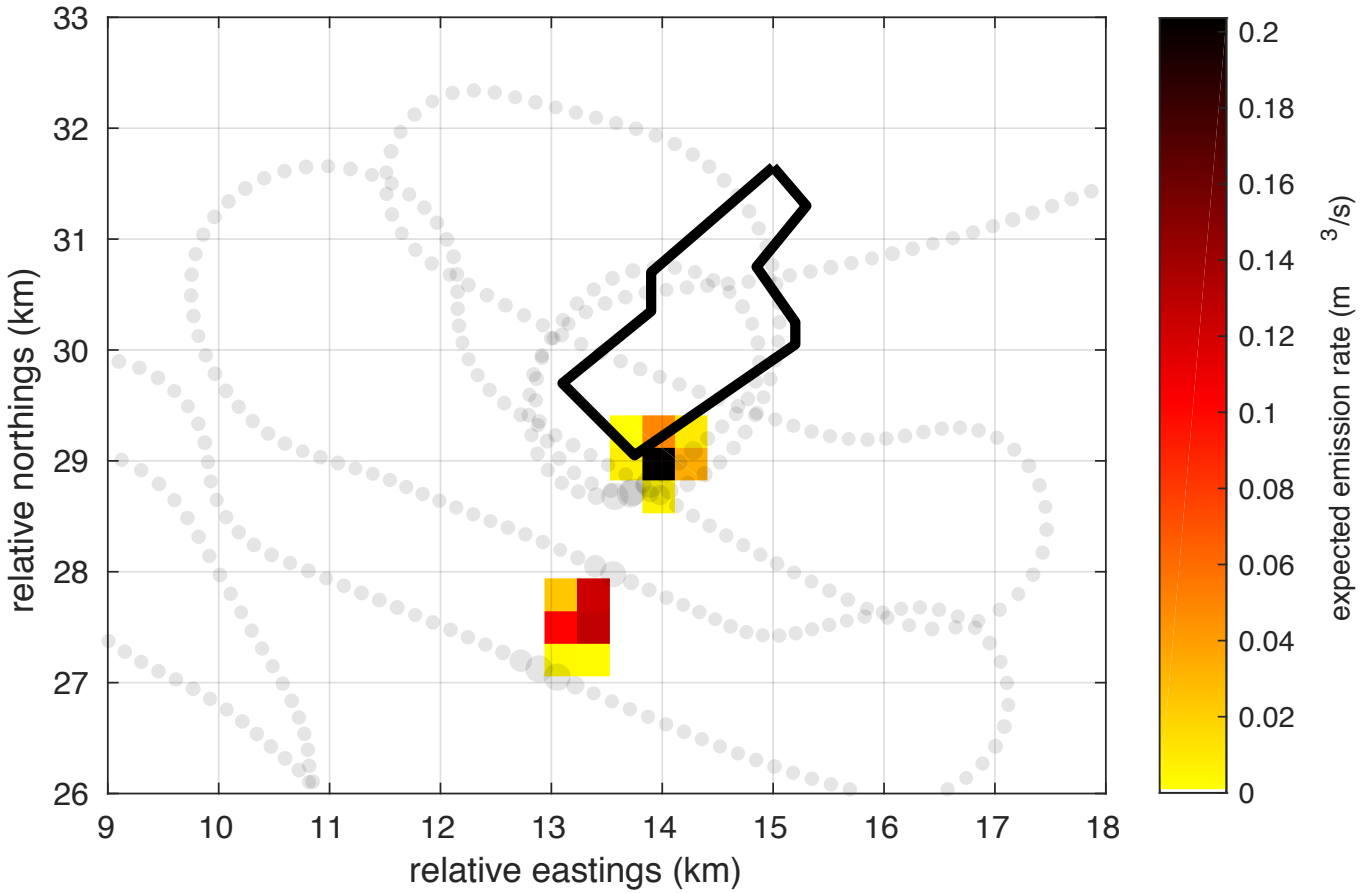
$$\mathbb{E}_{\boldsymbol{\theta}|\boldsymbol{y}}\left[f(\boldsymbol{\theta}) \mid \boldsymbol{y}\right] \approx \frac{1}{T}\sum_{t=1}^{T} f(\boldsymbol{\theta}^{(t)}) \qquad \text{where} \qquad \boldsymbol{\theta}^{(t)} \sim p(\boldsymbol{\theta} \mid \boldsymbol{y})$$

e.g. **Expected Probability of Event A (i.e. Marginal Distributions):** $f(\boldsymbol{\theta};\ A) = \mathbb{I}(\boldsymbol{\theta} \in A)$

Expected Emission Rate in Area $\mathcal{A}$: $f(\boldsymbol{\theta};\ \mathcal{A}) = \sum_k s_{kj} \int_{\mathcal{A}} \mathcal{N}(z;\ z_k, \mathbf{I}w_k^2)\,\mathrm{d}z$

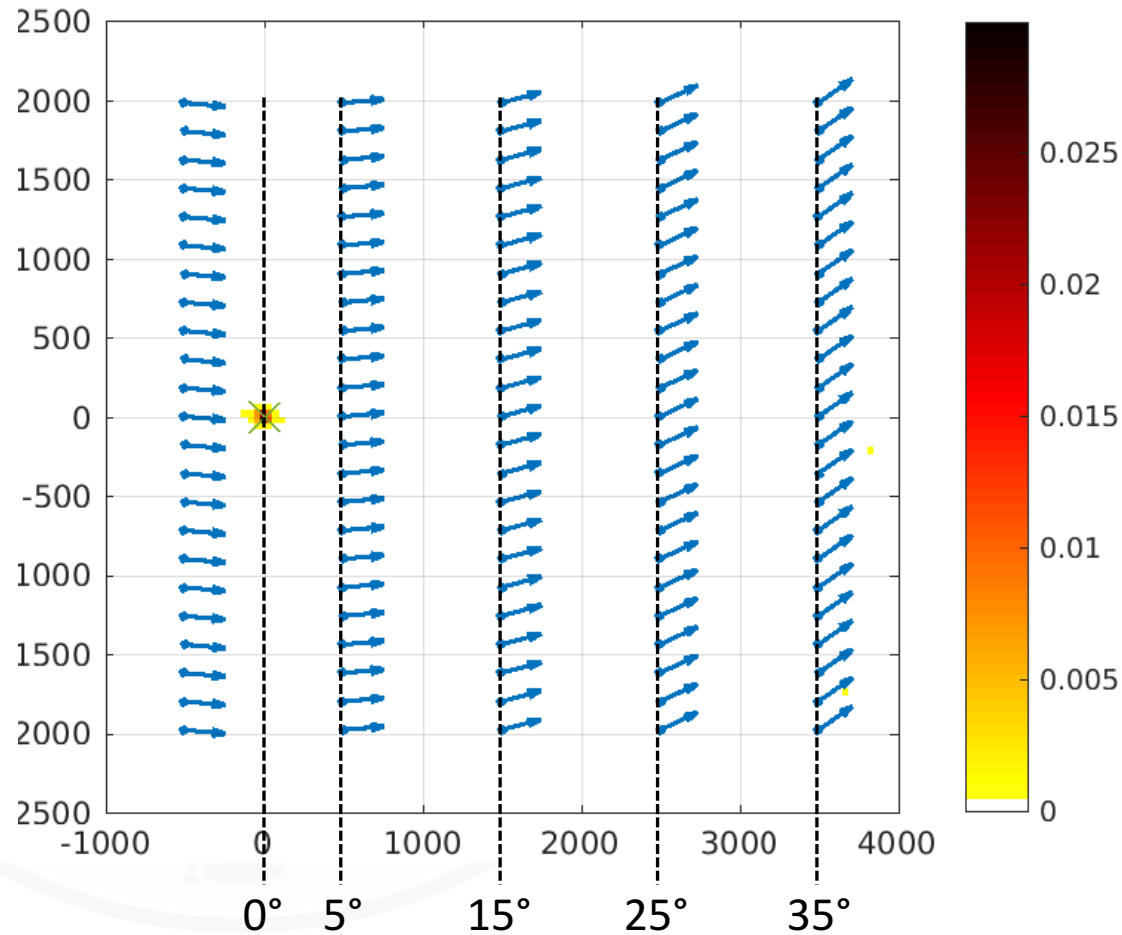
Expected Emission Rate

Expected Source Emissions Originating in Area $\mathcal{A}$: $f(\boldsymbol{\theta};\ \mathcal{A}) = \sum_k s_{kj} \int_{\mathcal{A}} \mathcal{N}(z;\ z_k, \mathbf{I}w_k^2)\,\mathrm{d}z$



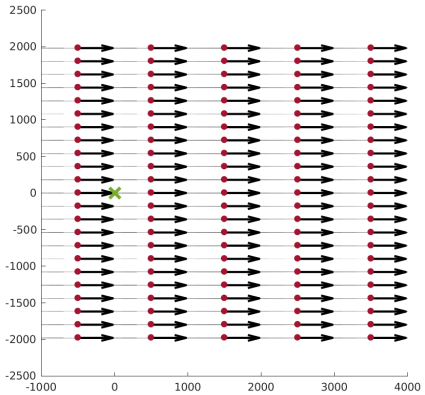
Synthetic Experiment: Linear vs. CPAB Wind Model

- Simulate observations using CPAB wind model.
- Compare inference using linear vs CPAB model.
- E.g. shift by 10°/1000 m.

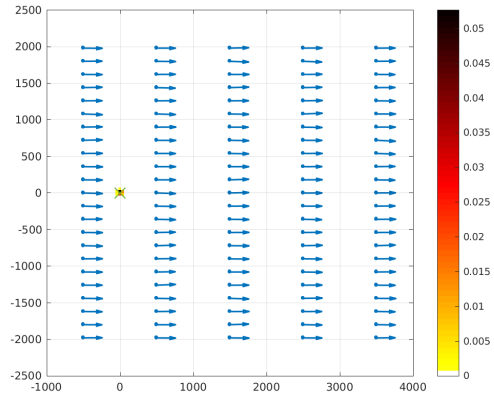


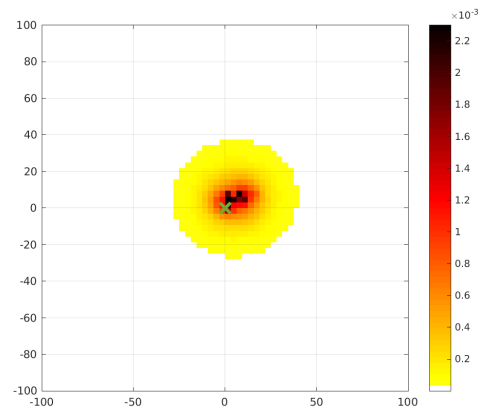
Synthetic Experiment: Angular Shift: 0° / km

Actual Flow Field

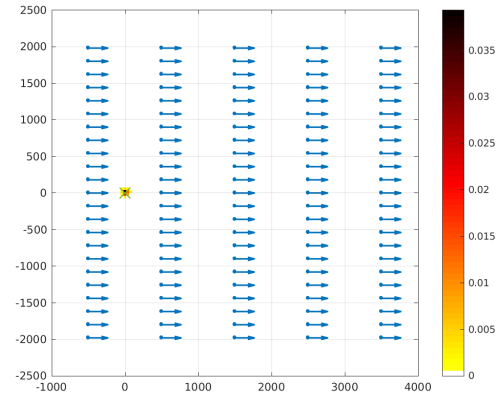


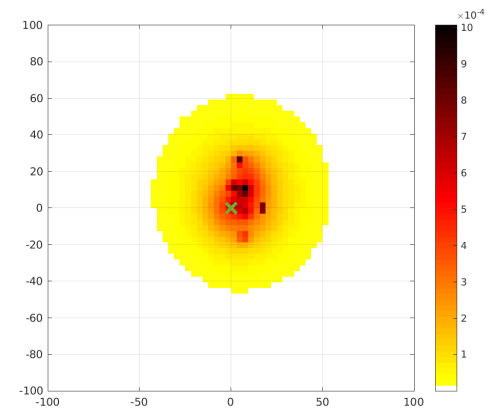
Linear Wind Model





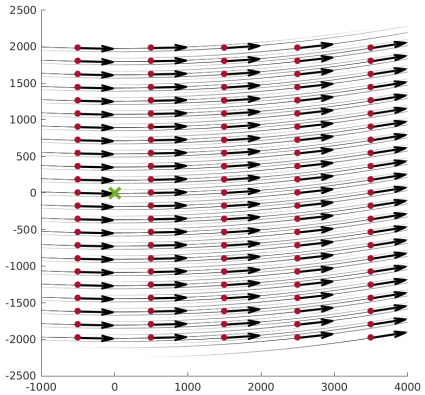
CPAB Wind Model



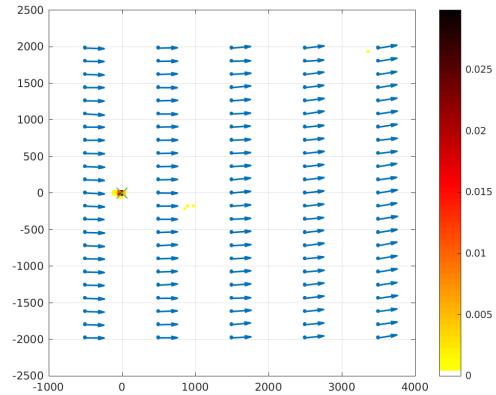


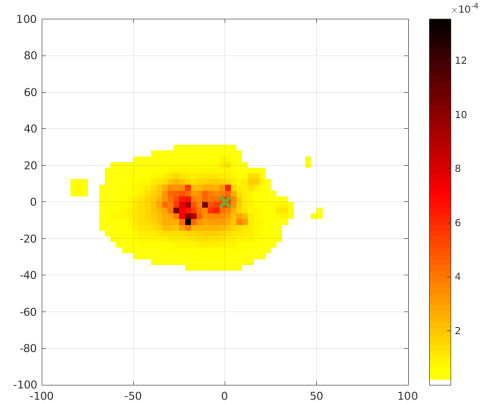
Synthetic Experiment: Angular Shift: 2.5° / km

Actual Flow Field

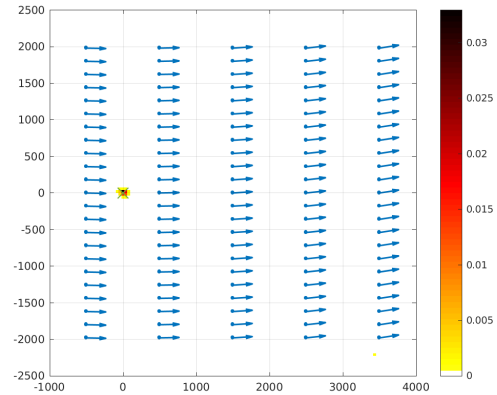


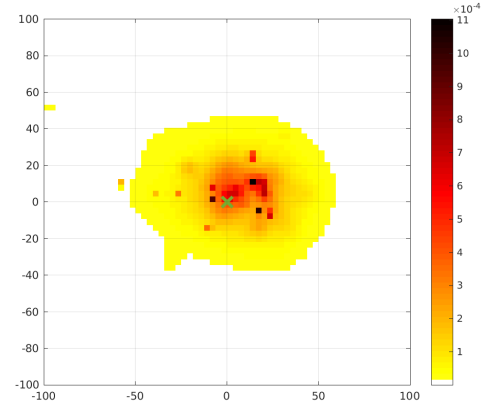
Linear Wind Model





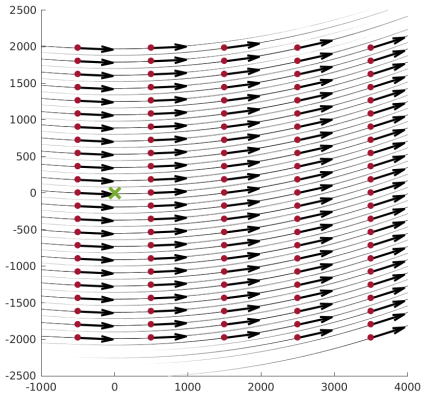
CPAB Wind Model



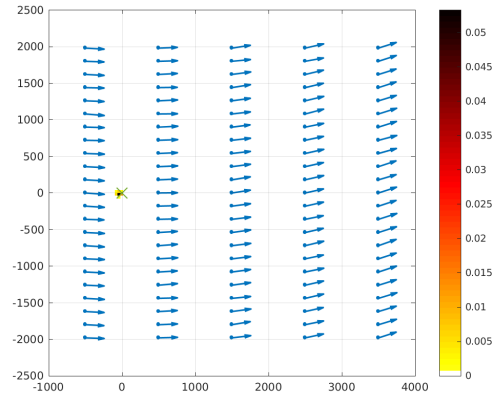


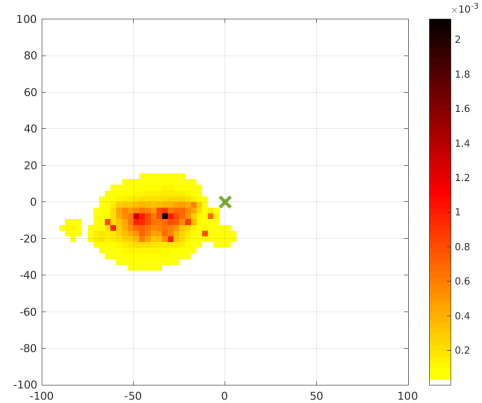
Synthetic Experiment: Angular Shift: 5° / km

Actual Flow Field

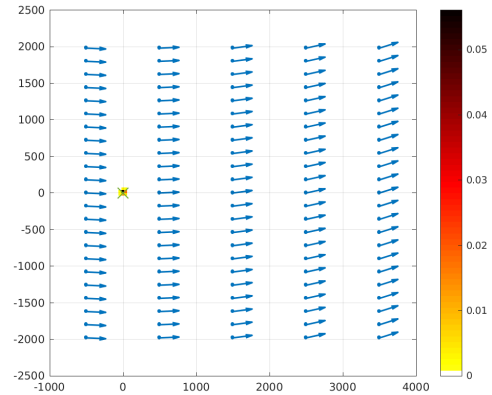


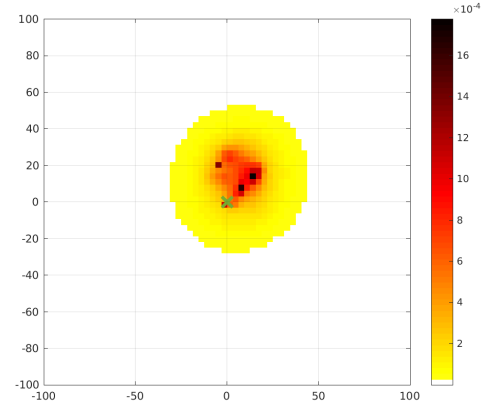
Linear Wind Model





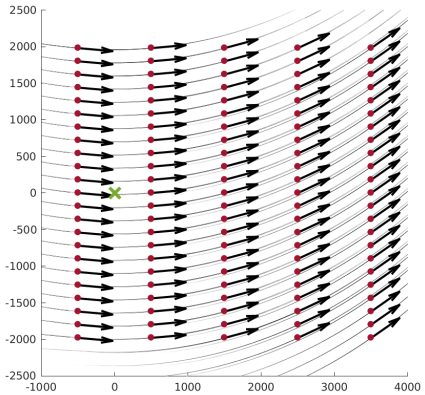
CPAB Wind Model



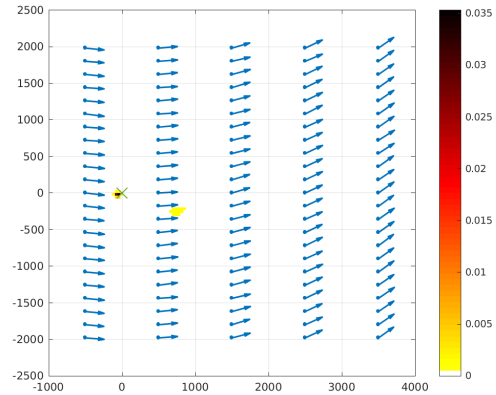


Synthetic Experiment: Angular Shift: 10° / km

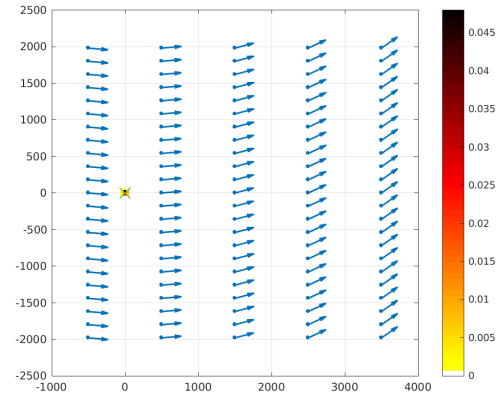
Actual Flow Field

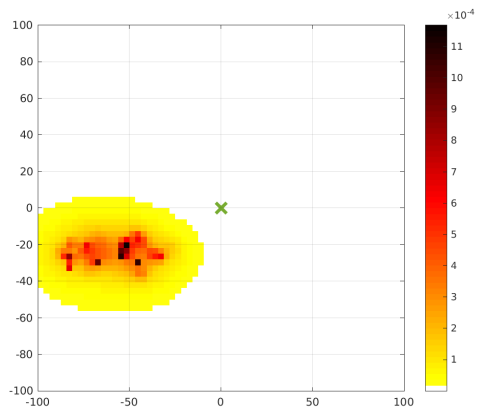


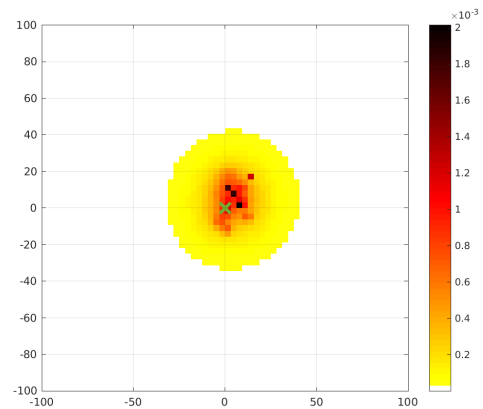
Linear Wind Model



CPAB Wind Model

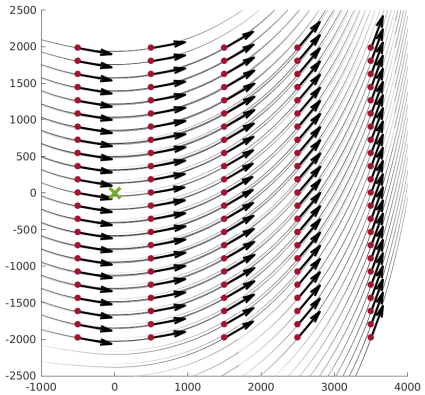




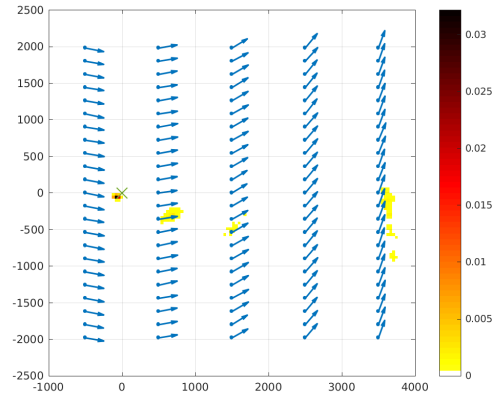


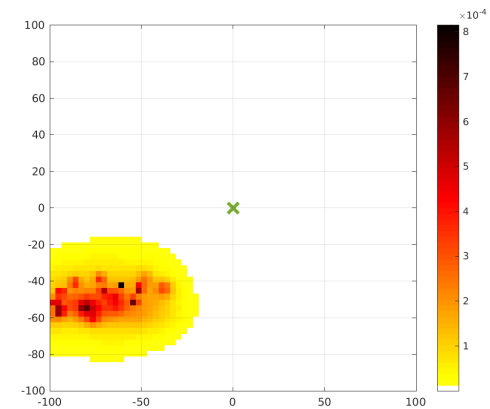
Synthetic Experiment: Angular Shift: 20° / km

Actual Flow Field

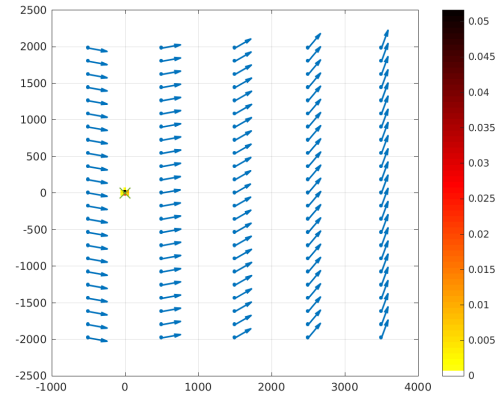


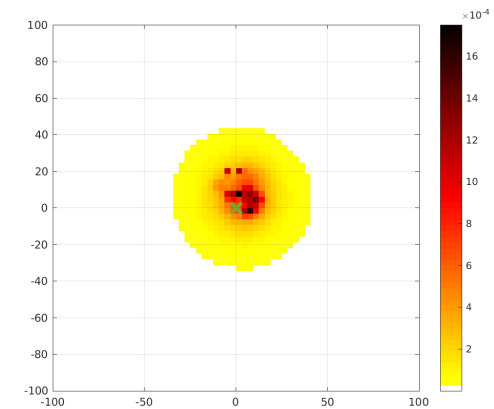
Linear Wind Model





CPAB Wind Model





Summary

- The CPAB wind model measurably improves localization accuracy over existing models.
- Sampling-based inference leads to efficient methods of evaluating system performance.
- We have demonstrated only a small subset of possible reasoning tasks over the posterior.
- This is a general purpose model that is applicable to many different material detection applications.

Comments & Questions

Bonus Slides

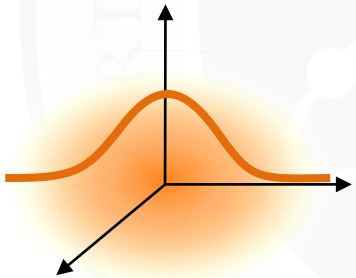
Model Description

Model of Emissions

We observe *continuous* releases from an unknown number of emission sources.

Each emission source is described by:

Emission Location

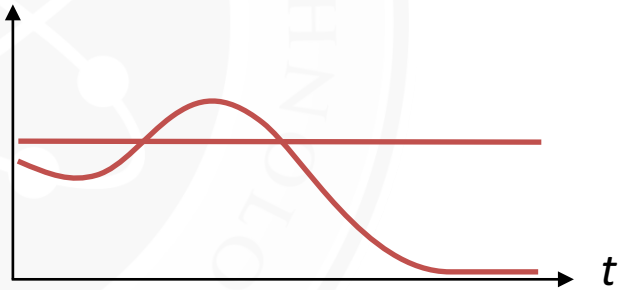


Normally-distributed with **mean** and isotropic **covariance** matrix

$$\mathcal{N}(z, Iw^2)$$

Emission Rate

(assumed **constant**)

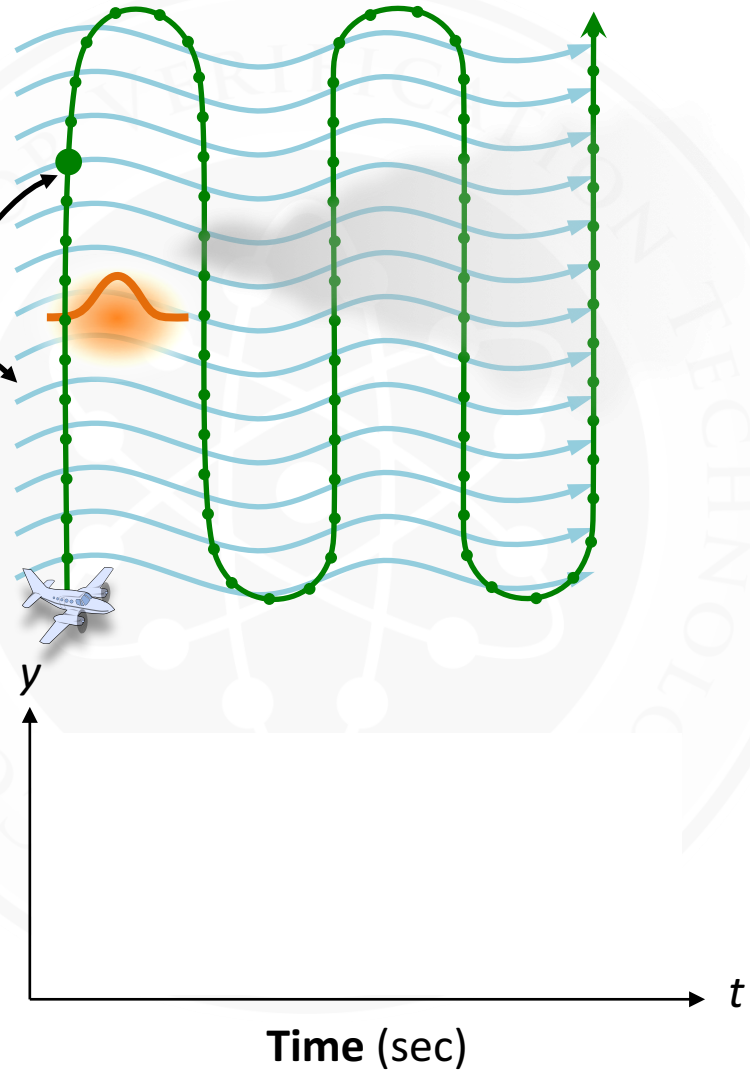


time (sec)

Data Collection

Each flight described by:

- **Concentrations** *y* at **times** *t* and **locations** *x*
- **Wind field** *U*
- **Measurement error variance** σ^2

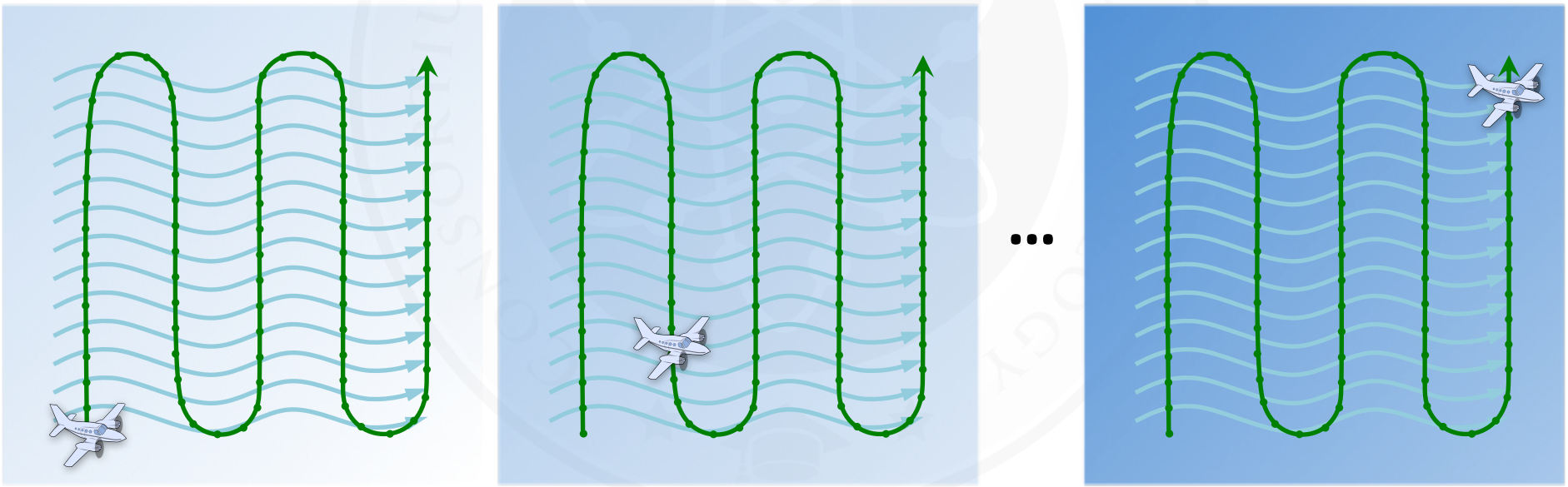


Concentration (ppb)

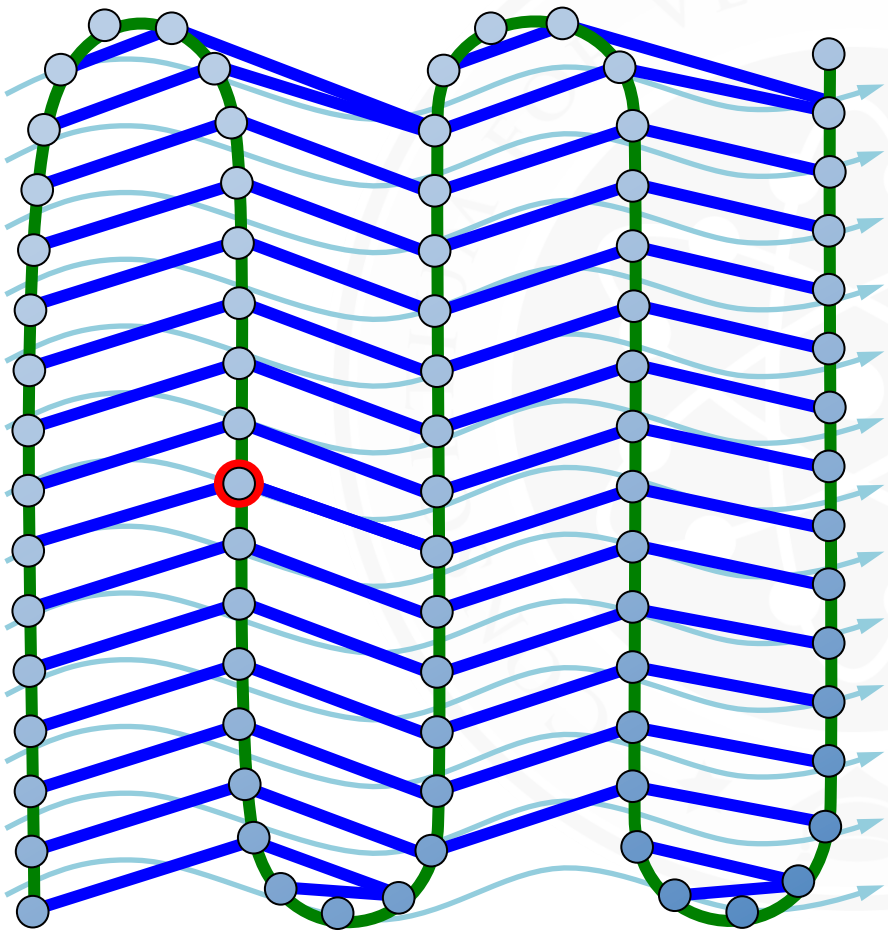
Ambient Substance Levels

- Our sensor captures both source emissions and spatio-temporally smooth **background concentrations**

t = 0 min **t = 10 min** **t = 30 min**



Gaussian Markov Random Fields



$$\{i,j\} \in \mathcal{E}$$

$$\updownarrow$$

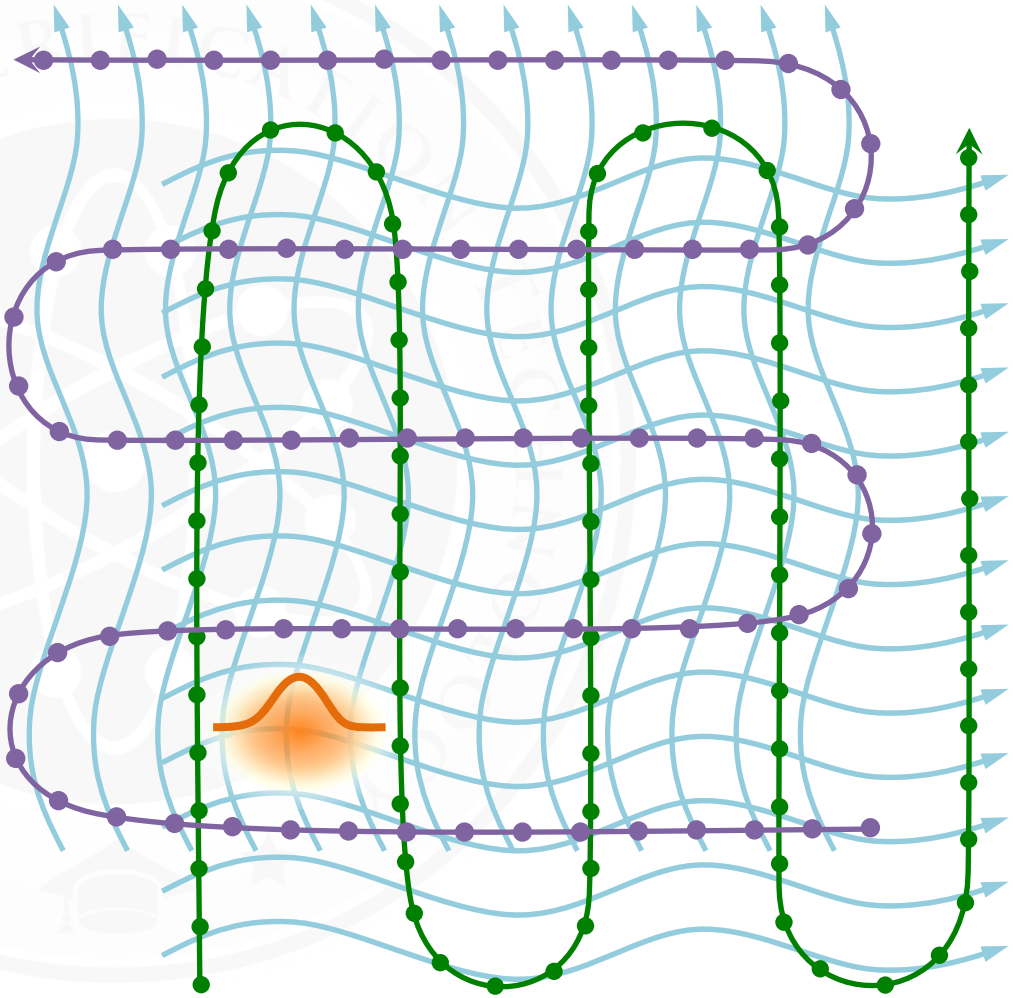
$$\Lambda_{ij} = \Lambda_{ji} = -\alpha_{ij} \neq 0$$

$$\alpha_{ij} = \frac{1}{\left(c_T|\Delta T_{ij}| + c_D \Delta D_{ij}\right)^2}$$

$$\Lambda_{ii} = -\sum_{j:j\sim i} \Lambda_{ij}$$

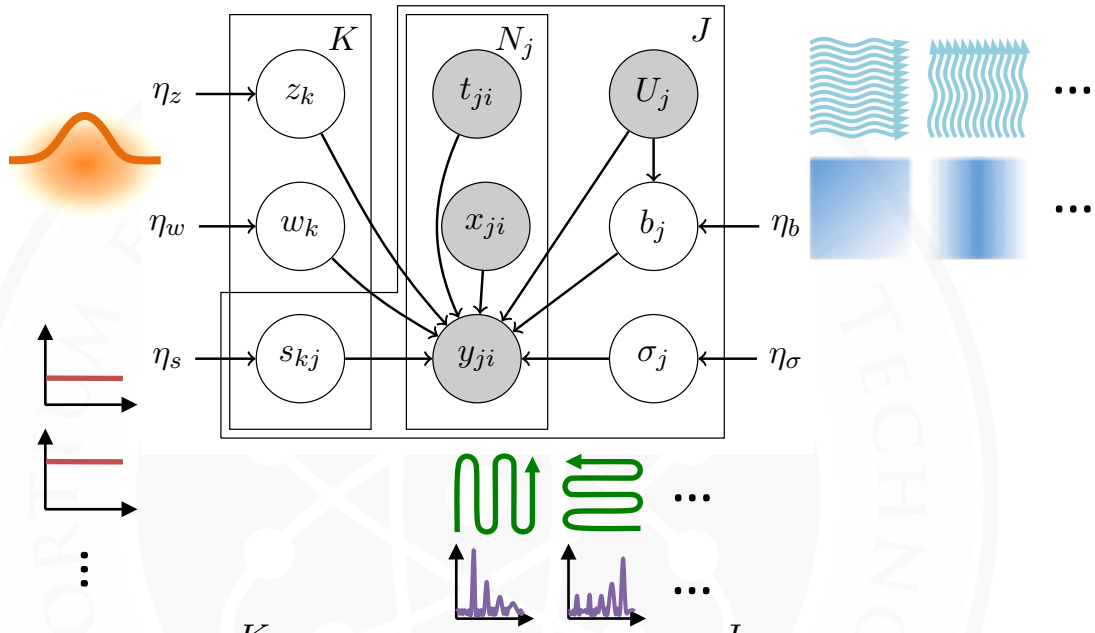
Multi-flight Surveys

We model emission rates and background fields as *virtually independent* from flight-to-flight.



Inference

Posterior Distribution



$$p(\boldsymbol{z}, \boldsymbol{w}, \boldsymbol{s}, \boldsymbol{b}, \boldsymbol{\sigma} \mid \boldsymbol{y};\ \boldsymbol{t}, \boldsymbol{x}, \boldsymbol{U}) \propto \prod_{k=1}^{K} p(z_k;\ \eta_z)\, p(w_k;\ \eta_w) \prod_{j=1}^{J} p(s_{kj};\ \eta_s) p(b_j;\ \eta_b)\, p(\sigma_j;\ \eta_\sigma)$$

$$\times \prod_{j=1}^{J} \prod_{i=1}^{N_j} p(y_{ji} \mid \boldsymbol{z}, \boldsymbol{w}, \boldsymbol{s}_{\cdot j}, b_{ji}, \sigma_j, t_{ji}, x_{ji}, U_j)$$

Posterior $\propto$ **Prior** $\times$ **Likelihood**

Why is Inference Hard?

- **Unknown number of sources!**
- Transport is uncertain
- Large number of latent variables
- Lack of closed-form analytical expressions
- Low signal-to-noise ratio

Inference Procedure

1. Obtain initial values via optimization
2. Use RJ-MCMC to draw samples of the posterior distribution
3. Use samples from the posterior distribution for analysis

Reversible-Jump MCMC

Birth:

Death:

Split:

Merge: